THE USE OF THE HISTORY OF MATHEMATICS IN TEACHING-LEARNING PROCESS: THE PERSPECTIVES OF FACULTY MEMBERS AND TEACHERS[1]

Abstract: The aim of this study was to investigate the faculty members' and the middle school mathematics teachers' perspectives regarding the use of the history of mathematics in the learning-teaching process of mathematics. As a phenomenological study, the qualitative data were collected through semi-structured interviews from 27 middle school mathematics teachers and seven faculty members and then, subjected to the content analysis. The findings revealed that both teachers and faculty members believed that using the history of mathematics is a worthwhile effort, with the potential to not only provide meaningful learning opportunities for students but also enrich teachers' professional development. However, it was also found that lack of historical perspective in the curriculum, teachers' inadequate knowledge, time constraint, no room for the history of mathematics in the textbooks and exams, overloaded curriculum and students' inadequate desire to learn were some of the reasons for rarely-use of the history of mathematics. Based on the overall findings of the study, it is concluded that teacher education (both pre-service and in-service), the structure of mathematics curriculum, teachers' and students' characteristics were the most important dynamics to integrate the history of mathematics into teaching effectively.

Keywords: the history of mathematics, mathematicians, mathematics teachers

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INTRODUCTION

Mathematics is a branch of science that manifests itself in every field from the smallest building blocks to the most complex formations in the universe. Striking reflections of mathematics can be seen in countless natural settings including the number of daisy flower petals, the intersecting spirals of pine seeds on cones, the elliptical orbits of the planets around the sun, the fixed value of division of the circumference by the diameter of all circles, or the helix curve shape of ivy around trees (Altun, 2008; Sertöz, 2011). Not only Galileo's description of nature as "a book written in the language of mathematics" (as cited in Topdemir, 2011, p.104) and also Sertöz's description mathematics as "creator's hints left in the nature" (2011, p.3) imply that mathematics has in fact existed in the universe since the genesis.

Since the first years of history, human beings started laying the foundations of mathematics in order to solve the daily needs, encountered problems, and to understand the universe. The earliest mathematical activity started approximately 5000 B.C. around Egypt, Mesopotamia, China, and India (Baki, 2014; Bell, 1992). The discipline gradually flourished as number systems were invented by the Babylonians, the Egyptians, the Chinese, and the Greeks fulfilling their daily needs; mathematical and arithmetic studies were carried out by mathematicians such as Pythagoras and Euclide; and Plato performed studies on irrational numbers and smooth polyhedrons (Abdulhay, 2014; Baki, 2014; Cajori, 1919/2014). In the following years, significant progress were made in the finest calculation of trigonometric values, in algebra, cubic equations, and logarithm, derivatives, integral, and complex numbers, eventually resulting in an exponential accumulation of mathematical knowledge (Baki, 2014; Cajori, 1919/2014; Zeki Bey, as cited in Demir, 2004). After the 19th century, developments in mathematics have spread across the world; as a result, significant numbers of mathematicians in different parts of the world have contributed to mathematics in various ways.

In the 21st century, understanding of mathematics has of utmost importance since learning mathematics provides individuals to develop scientific thinking skills, apply them to different situations, produce original ideas, make research, gain self-regulation skills, and develop self-confidence (Hattatoğlu, 2010; Işık, Çiltaş & Bekdemir, 2008; Karakurumer, 2003; National Council of Teachers of Mathematics, 2000; Rizki & Priatna, 2019). Putting emphasis on the essence of mathematics literacy, Ojose (2011, p.91) stated that "Mathematics is so entwined with today's way of life that we cannot fully comprehend the information that surrounds us without a basic understanding of mathematical ideas". According to the framework of the Programme for International Student Assessment (PISA) 2021, mathematical literacy is "an individual's capacity to reason mathematically and to formulate, employ, and interpret mathematics to solve problems in a variety of real world contexts. … It assists individuals to know the role that mathematics plays in the world …" (OECD, 2018; p.7). Tekin and Tekin (2004) also argued that mathematically literate person has knowledge and skills about the following areas: (a) content area literacy; (b) mathematical thinking process; (c) up-to-date knowledge of mathematics, and (d) historical evolution of mathematics. In a similar vein, Steen (2001) acknowledged "Cultural Appreciation" as one of the fundamental elements for the concept of quantitative literacy, and defined as "Understanding the nature and history of mathematics, its role in scientific inquiry and technological progress, and its importance for comprehending issues in the public realm" (p.8). In this respect, being mathematically literate requires understanding of the historical side of mathematics including not only evolution of mathematics as a discipline but also the leading mathematicians and their contributions. Comparing the perspectives from past to present, Ding (2019) also emphasized the essence of the history of mathematics as follow:

> "In the past, it was generally believed that the study of mathematics history was an elegant, sunny and white snow-like model. However, in today's era, mathematics teaching … pays more attention to the cultivation of ideological methods and emotional attitudes and values. The history of mathematics has thus got rid of the situation … and it has gradually been valued …" (p.783)

The history of mathematics is a field of study that put forwards past obstacles and difficulties which mathematicians have overcome in the development of mathematics; reveals mathematics' dynamic nature (Liu, 2003) and "shows the evolutionary and progress of mathematical knowledge through civilizations" (Baki, 2014, p.3). In other words, history of mathematics is a comprehensive area that deals with the growth

processes of mathematics, the lives, works, achievements or failures of leading figures who have contributed to mathematics, the social and cultural dimension of mathematics, and development and progression of mathematical knowledge (Bidwell, 1993; Burton, 2003; Eves, 1990; Katz, 1993; Otte, 2007; Pepe, & Guerraggio, 2017; Yee and Chapman, 2011). Studies on the use of history of mathematics in mathematics education first appeared in the 1970s. The research in this area is now being supported by various world-wide institutions and organizations concerned with mathematics education (e.g. the International Commission on Mathematical Instruction [ICMI]) (Clark, Kjeldsen, Schorcht, Tzanakis and Wang, 2016; Fauvel and Maanen, 2002; Fried, 2001).

The findings of various research studies pointed out that the use of history of mathematics in learning and teaching process bears potential contributions to both students and teachers. Specifically, it is stated that the history of mathematics helps students to comprehend the formation of mathematical thinking, improve problem-solving skills, assess mathematical topics in a comparative way between the past and present, establish relationships between mathematical topics and other disciplines, and appreciate that mathematics is a constantly evolving discipline (Alpaslan, 2011; Ho, 2008; Jankvist, 2009; Lim & Chapman, 2015; Liu, 2003; Sullivan, 1985; Wilson and Chauvot, 2000). Besides, the history of mathematics has a supporting role for teachers to gain different perspectives, to comprehend mathematical facts unnoticed before, and to move from product-oriented instruction to process-oriented (Radford, 2014). Teachers, while blending their qualified knowledge about the history of mathematics with in-class activities, can develop their creativity and also acknowledge the reason for teaching each specific topic. As a result, their teaching skills might improve (Furinghetti, 1997; Guillemette, 2017; Haile, 2008; Kjeldsen, 2011; Liu, 2003; Nataraj & Thomas, 2009; Pengelley; 2002). Bidwell (1993, p.461) notes that students think mathematics "as a closed, dead, and emotionless island; where teachers can rescue them for replacing them on an alive, open, full of emotion, and always interesting mainland" when they integrate history of mathematics in the learning and teaching of mathematics. In the literature, there have been numerous studies emphasizing that the integration of mathematics history into the learning and teaching process might provide more meaningful and real-life connected learning environment, yet there have been some factors affecting the development of such environment. According to the literature, such issues as no room for the history of mathematics in the curriculum, lack of instructional resources and/or materials, teachers' limited or lack of knowledge about the history of mathematics, etc. (Baki & Yıldız, 2010; 2016; Başıbüyük, & Şahin, 2019; Sözen, 2013; Fried, 2001; Siu, 2007; Niitsuma & Nagaoka, 2014; Tan-Şişman & Kirez, 2018).

Although there have been various research studies conducted with mathematics teachers on why's and how's of the mathematics history, to our knowledge, there is no research study on the views of mathematicians regarding the use of the history of mathematics. Ding (2019) argued that neither theory (the pure knowledge of mathematics history) nor practice (the pure knowledge of how to teach mathematics) has produced better ways to integrate the history of mathematics. In this respect, the purpose of the present study was to investigate the views of faculty members' and mathematics teachers' regarding the use the history of mathematics in the learning-teaching process of mathematics. More specifically the study seeks to answer the following research questions:

1. What are the faculty members' opinions about the use of the history of mathematics in learning and teaching process?
2. What are the the mathematics teachers' opinions about the use of the history of mathematics in learning and teaching process?

It is believed that portraying comparatively the views of faculty members, who have deep theoretical knowledge of the mathematics discipline, and the mathematics teachers, who are practitioners of the learning-teaching process, about why and how to use the history of mathematics is essential to promote theory-enhanced practice for the integration of the history of mathematics. It is also expected that the results of this study will be valuable for many stakeholders. Firstly, the findings will contribute to mathematics teachers who are one of key actors in interpreting and transforming written curriculum into the learning and teaching process. In addition to the mathematics teachers, the findings of this study may also be worthwhile for curriculum developers to provide comprehensive bases of why and how the history of mathematics should be used in mathematics education in terms of the different points of view. Besides, this study might

provide helpful insights and implications, emerged from not only the experiences of the mathematics teachers but also from the suggestions of the faculty members with their in-depth content knowledge, both for curriculum development and implementation process.

METHOD

RESEARCH DESIGN

In this qualitative study, the phenomenological approach that focuses on "to seek reality from individuals' narratives of their experiences and feelings, and to produce in-depth descriptions of the phenomenon" (Yüksel & Yıldırım, 2015, p.1) was used. Since the focus of this study was on the insights of faculty members and mathematics teachers about why and how the history of mathematics should be used in teaching and learning process; the phenomenological research design was employed to understand how a phenomenon, which in this case was the use of mathematics history in learning and teaching processes, comprehended among different stakeholders.

PARTICIPANTS

The study was conducted with seven faculty members working at different Turkish public universities and 27 mathematics teachers working at the public middle schools located in six central districts of Ankara. The faculty members were selected by convenient sampling. Through searching on the official web pages of the Turkish state universities, the researchers listed the faculty members who were working as a full-time faculty at the departments of mathematics or mathematics education and were interested in the history of mathematics. Then, the faculty members were selected on the basis of their accessibility and convenience to the researchers and they were invited to the study via e-mail. As given in Table 1, totally seven faculty members from five different state universities volunteered to take part in this study. Among seven faculty members, four of them were working as a full-time faculty at the department of mathematics education and three were at the department of mathematics. With regard to the K-12 level teaching experience, three of them have experience of 1-5 years, three have no experience and one has experience of 5-10 years. In addition, while the majority of the faculty members (n=6) did not take any course about the history of mathematics during their undergraduate or graduate education, four of them lectured the history of mathematics course at undergraduate and/or graduate level.

Table 1. The Faculty Members' Profiles

Faculty Members' Profiles	F1	F2	F3	F4	F5	F6	F7
Gender	Male	Male	Male	Male	Male	Male	Male
Age	40-45	61-65	50-55	50-55	50-55	55-60	61-65
Title	Asst. Prof. Dr.	Assoc. Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.	Prof. Dr.
University	A	B	C	D	B	E	C
Department	Math	Math Ed.	Math Ed.	Math	Math Ed.	Math Ed.	Math
K-12 teaching experience	-	5-10	1-5	-	1-5	1-5	-
Taking HoM* course	No	No	No	No	No	Yes	No
Lecturing HoM course	No	Yes	No	No	Yes	Yes	Yes

*HoM: The history of mathematics

Further, the mathematics teachers were selected through maximum variation sampling method that allows researchers to collect in-depth information and mirror divergent perspectives rather than making a generalization (LeCompte, Preissle, & Tesch, 1993; Patton, 2015). In this scope, 27 math teachers from 12 different middle schools located in Çankaya, Yenimahalle, Etimesgut, Keçiören, Pursaklar, and Altındağ districts of Ankara took part on a voluntary basis. Besides, the central district where the school located, the population of the schools, the types of schools (double-shift [DS]/single-shift [SS]), years of experience in teaching, and educational background was taken into account while determining the participants. As seen

in Table 2, 23 female and four male mathematics teachers were participated to the study. With regard to teachers' years of experience in teaching, it ranges from one to 35 years, while the majority (*n*=9) has experience of 1-5 years. In addition, 22 of the participants graduated from faculty of education and five from faculty of science.

Table 2. The Mathematics Teachers' Profiles

Teachers	Gender	Graduation	District	Teaching experience	School population	Type of schooling
T1	Male	F.ED*	Keçiören	26 and above	1000-1499	DS
T2	Female	F.ED	Keçiören	11-15	1000-1499	DS
T3	Female	F.ED	Etimesgut	11-15	1500-1999	DS
T4	Female	F.ED	Altındağ	1-5	1000-1499	DS
T5	Female	F.ED	Yenimahalle	11-15	1000-1499	DS
T6	Female	F.ED	Pursaklar	1-5	1000-1499	DS
T7	Female	F.ED	Etimesgut	6-10	500-999	SS
T8	Female	F.ED	Etimesgut	16-20	500-999	SS
T9	Female	F.ED	Yenimahalle	11-15	1500-1999	DS
T10	Female	F.ED	Altındağ	1-5	500-999	DS
T11	Male	F.SC**	Yenimahalle	16-20	1000-1499	DS
T12	Female	F.ED	Keçiören	1-5	1000-1499	DS
T13	Female	F.SC	Keçiören	26 and above	1000-1499	DS
T14	Female	F.ED	Altındağ	1-5	500-999	DS
T15	Female	F.ED	Pursaklar	1-5	1500-1999	DS
T16	Female	F.SC	Altındağ	1-5	1000-1499	DS
T17	Female	F.ED	Çankaya	6-10	500-999	DS
T18	Female	F.ED	Pursaklar	1-5	1500-1999	DS
T19	Female	F.ED	Çankaya	11-15	500-999	SS
T20	Female	F.ED	Yenimahalle	11-15	1000-1499	DS
T21	Female	F.SC	Yenimahalle	21-25	1500-1999	DS
T22	Female	F.ED	Pursaklar	11-15	1500-1999	DS
T23	Male	F.ED	Çankaya	21-25	500-999	SS
T24	Female	F.ED	Çankaya	16-20	500-999	SS
T25	Female	F.SC	Çankaya	16-20	500-999	SS
T26	Female	F.ED	Yenimahalle	6-10	1500-1999	DS
T27	Female	F.ED	Pursaklar	1-5	1000-1499	DS

F.ED*: Faculty of Education F.SC**: Faculty of Science

DATA COLLECTION INSTRUMENTS

In this study, Faculty Members Interview Form (FMIF) and Teachers Interview Form (TIF) were developed by the researchers and used as the main data collection instruments. While developing the interview schedules, first of all the literature was reviewed in detail to outline the important issues regarding the use of the history of mathematics in teaching and learning process. Through synthesizing the information gathered from the literature, the draft interview schedules were written in line with the research questions. Afterwards, the drafts went through revision with the help of expert opinions gathered from two faculty members from the department of Educational Sciences and two mathematics teachers. Based on feedbacks received from the experts, the wording of some questions was changed and some follow-up questions and prompts were either added or removed in order to make questions more clear and understandable. Then, the draft TIF was piloted with three mathematics teachers and FMIF was piloted with two faculty members to determine whether the questions make sense to the interviewees and as a result, no changes were applied to the drafts. In this respect, the final interview schedules were composed of the questions about demographic information (e.g. age, educational background, etc.) and the use of history of mathematics in learning and teaching process (e.g. "What is your opinion about the integration of the history of mathematics in teaching and learning of mathematics?", "What do you think about the current situation of the use of the history of mathematics in classrooms?", "What should be done to use the history of mathematics effectively?").

DATA COLLECTION AND ANALYSIS

Before the collection of the data, the necessary permissions from the Ethics Commission of Hacettepe University and the Ministry of National Education were obtained. In order to conduct interviews with mathematics teachers, the researchers contacted with the principals of the selected schools for scheduling the short meetings to introduce the study. Then, the interviews were conducted with 27 mathematics teachers who had volunteered to participate in the study. The faculty members were invited to the study via e-mail and totally seven faculty members from five different state universities volunteered to take part in the study. All interviews were audio recorded and lasted approximately between 30 to 45 minutes.

The collected data were subjected to content analysis. The steps followed during the content analysis process were: (1) transcribing the raw data, (2) organizing and preparing data for analysis, (3) reading through all data, (4) coding the data, (5) generating themes or description, (6) interrelating themes/description and (7) interpreting the meaning of themes/descriptions (Creswell, 2013; Yıldırım & Şimşek, 2013). In this respect, first of all, the interviews were transcribed word by word on a word document. Then transcribed data were read thoroughly for a few times considering the research problems to identify the conceptual framework. The coding was done by considering approximately every expression used by participants not missing any important details. Finally, the codes focusing/implying the similar ideas were combined under categories and themes. The teachers were coded as T1 through T27; while the faculty members were represented with F1 to F7. The opinions of the participants were reported with direct quotations.

TRUSTWORTHINESS

Trustworthiness of a research is characterized as provisions or actions that establish reliability and persuade readers about accuracy of the findings (Lincoln & Guba, 1985). To enhance trustworthiness of the present study, several strategies were used such as credibility, transferability, dependability, and confirmability. To establish credibility, triangulation via data sources (Shenton, 2004) was carried out among the teachers' and field experts' perspectives. The codes in data analysis were submitted to one faculty member from the department of Educational Sciences, who have knowledge of the research problem and qualitative research methods, as required by peer debriefing for credibility. In addition, interviews were recorded with a recording device with participants' consent and the recordings were verified by participants at the end of the interviews for member checks. Further in order to help transferability of the study, the scope and limitations of this research were set out clearly and the methodology of the study was explained thoroughly. Besides, the data were enriched by maintaining quotes from participants as well as displayed in details in order to provide thick description for the sake of trustworthiness of the study. During the study confirmability, namely "the qualitative investigator's comparable concern to objectivity" (Shenton, 2004; p.72), were provided by making use of audit trail and triangulation. The detailed methodological descriptions were made to allow integrity of research results to be investigated. In addition, the data were reported by reducing the effect of researcher bias. Lastly, Miles and Huberman's (1994) formula (Reliability = Number of agreements / (Agreements + Disagreements) x 100) was used for reliability analysis. In this respect, two randomly selected transcriptions of the participants (two from teachers, two from faculty members) were coded by one researcher and one expert separately and independently. The agreement between coders was found as .87 for TIF and .84 for FMIF. To solve the disagreements, both sets of data were compared, and through discussion, the disparities were reconciled to reach a consensus.

RESULTS

THE VIEWS OF THE FACULTY MEMBERS REGARDING THE USE OF HISTORY OF MATHEMATICS

The findings revealed that all of the faculty members expressed that the history of mathematics should be used in learning and teaching process. They believed that students could benefit from using the history of mathematics in the classroom as it provides opportunity to (a) internalize the idea that mathematics is a human creation, (b) enhance their learning, (c) understand the process of mathematical knowledge formation, (d) gain a rich perspective on mathematics and (e) appreciation of mathematics. In addition, the

faculty members stated that making use of the history of mathematics might help for mathematics teachers to enhance their content knowledge and support to implement student-centered instruction. In the Table 3, the results were summarized with quotations from the faculty members' views.

Table 3. The Benefits of Use the History of Mathematics

Benefits for Students	*f*	*Faculty Members' Views*
Internalizing the idea that mathematics is a human creation	3	*"…Most of the students leave the lessons with a wrong impression like this: 'What I've learned right now … is came out of the minds of a mathematician or a few mathematicians … as a whole like the way the teacher taught me'. In fact, what one needs to know what stages have been through for bearing that theorem, its simple forms, the issues inspired that theorem."* F7
Enhance students' learning	2	"*…According to the Fuzzy logic, there are gray areas between zero and one. You need to rate them, too. So it's been a lot of application to technology. For example, dishwashers, you set the program as very dirty, less dirty or even lesser dirty… That's Fuzzy logic! Now, if this kind of information is given during the lesson or on textbook or a teacher draws students' attention by mentioning them in two or three minutes; s/he can transform abstract structure of mathematics into concrete one."* F5
Understanding the process of mathematical knowledge formation	1	"*You know, students always ask about mathematics 'What does that do? Where does it come from?' It is taught in a very abstract way. When it is taught with the historical perspective, students can be able to see that in reality, every concept was born by a need, or this need may be both the need of physics and math or came as a need in mathematics itself, but nothing contains an abstract nature in the sense that students think. Abstract but that abstraction surely solves a problem responding to something. That is the biggest advantage of the historical perspective."* F4
Gaining a rich perspective of mathematics	1	"*One day I asked; "Who is this Binomial, what is the nationality?" etc. Some of the students said German, some said British. They think Binomial is a mathematician! There is no mathematician called Binomial!... Then I felt that we should look at the history of mathematics. For instance, the Pythagoras Theorem, who is Pythagoras? Is Pythagoras a human? etc. Instead of focusing only the memorization of the theories, we need to provide students with a deep and broad view of mathematics.*" F2
Appreciation of mathematics	1	"*No matter which country you go to around the world, you see that mathematics and mathematicians are always perceived differently. Mathematics is always regarded as difficult, like in our country, but doing math is considered as the indicator of a much higher standard. It is written in many articles, mathematics has a door opener position for professional career, it is true for not only in our country, but for the whole world. This means that through the history of mathematics, students can realize that how mathematics is valuable and important*." F5
Benefits for Teacher	*f*	*Faculty Members' Views*
Enhancing the content knowledge	1	"*The strength of the use of history of mathematics in terms of a teacher, s/he can teach better. Because s/he gains knowledge about where the concepts come from, how the problems are related to.* "F4
Supporting student-centered instruction	1	"*The history of mathematics is one of the best medium of instruction for the teachers who would like to design student-centered learning environment; by making use of the examples chosen from the history of mathematics, s/he might group students to work together and implement problem solving method*." F6

Some of the faculty members also mentioned such restricting-issues related to the use of history of mathematics as "loss of prestige and trust" and "extra workload" due to teachers' poor/lack of knowledge. In Table 4, the results were summarized with quotations from the faculty members' views.

Table 4. The Restricting-issues about Use of the History of Mathematics

Teacher-related issues	*f*	*Faculty Members' Views*
Loss of prestige and trust	2	"*While teaching Binominal Theorem, one of the students asks, 'When did Binomial live, my teacher?' if the teacher doesn't know, s/he is ruined there! Ruined!"* F2
Extra workload	1	"*The history of mathematics is a completely different area. A math teacher is not normally expected to know too much about the history of mathematics... It is an additional burden for her/him to deal with the history of mathematics… Spending extra efforts on it spending more time on..."* F4

Further, the results indicated that all faculty members thought that the history of mathematics is almost never integrated in current mathematics lessons due to several reasons as summarized in Table 5.

Table 5. The Reasons for the Rarely-use of the History of Mathematics

Reasons	*f*	*Faculty Members' Views*
Teachers' inadequate knowledge about the history of math	4	*"Since the teacher did not get such training, s/he has not any experience or accumulated knowledge of it (the history of math), the teacher is the biggest obstacle already. S/he is not trained on that ... the most crucial thing is that the teacher is at that depth; aware of that work. First s/he is introduced to the use of it (the history of math) become familiar with the activities then, s/he is able to use of it when becomes a teacher in the future*." F6
		"*Of course, s/he (a teacher) needs to have adequate knowledge of the developmental journey of mathematics. In fact, if s/he has enough knowledge, s/he automatically starts to talk about it at least for two minutes. So s/he says it anyway. However, if a teacher cannot have enough knowledge, s/he cannot reflect it. The teacher is particularly important here*." F5
Exam-oriented education	3	"*Above all, there are exams. As long as these exams are here, it (the history of math) will be like an additional burden for students because the goal is to get ready for the exams not to learn mathematics at all. It is very difficult to use the history of mathematics in such a system, we have to accept it*." F4
MoNE's uninterested approach	2	"*MoNE should understand the importance of it (the use of the history of math). In order to do this, a lot of work has to be done. We need to explain the importance of the history of mathematics to them (MoNE). There is also an obstacle as well. For instance one is that the decision makers are not aware of the importance of the issue enough and also they underestimate how to integrate this issue into education and class. They think that it will be done as one says 'Now do this!' but it is not the case in the class indeed." F3*
Overloaded curriculum	1	"*Teachers are required to follow and complete the curriculum but one can use the history of mathematics if s/he wants. It looks like the main obstacle*." F5
Lack of historical perspective in the curriculum	1	"*Well, there's no obstacle. Indeed, there is no room for it (the history of math) in the curriculum, so it is not reflected. But if the curriculum includes it, I guess that our students will do it with pleasure. Just put it in, I don't know why it is not taken into consideration, it must be included in the curriculum. Because it is really needed. Look, we recall Pythagoras, but the student should be able to say "Pythagoras was someone, who like that and did something, happened before Christ" and like that. For example, we always call the Euclidean Theorem. Who is this Euclid, isn't it? Here we call the Euclidean Geometry. Well, he must be mentioned in geometry, if not it is incomplete*." F2
Teachers' extra responsibilities	1	"*The man (teacher) has the concern of subsistence, does not earn enough money, plus, there is a lot of burden on him, then he is educator... when come to school, he has to deal with such issues as making a lesson plans, activities, attending meetings and so on. So teacher is already under very big burden. If there is no burden and extra loads, it (the use of history of math) would be possible. However, it's been a big problem since the past, and I don't think it's overcome at the moment*." F7

Lastly, the faculty members proposed the ways to effectively integrate the history of mathematics in the teaching and learning process. Their recommendations, as presented in Table 6, focused on mathematics curriculum, and teacher education.

Table 6. The Faculty Members' Recommendations for Effective Use of History of Mathematics

Faculty Members' Recommendations	*f*
Inclusion of the history of mathematics in the curriculum	4
Organizing student-centered learning environment	3
Inclusion of the history of mathematics course in pre-service teacher education program	2
Providing in-service training programs	2
Preparing instructional materials and resources about the history of math	1
Inviting more faculty members having deep knowledge about the history of math to the curriculum development commissions	1

The Views Of The Mathematics Teachers Regarding The Use Of History Of Mathematics

The findings revealed that apart from one teacher, all of the mathematics teachers expressed that the history of mathematics should be used in the learning and teaching process. As given in Table 7, the teachers stated that using the history of mathematics was a worthwhile effort, with the potential to not only provide meaningful learning opportunities for students and also enrich teachers' professional development. Besides,

the mathematics teachers stated that making use of the history of mathematics might contribute to the teaching process of mathematics.

Table 7. The Benefits of Using of the History of Mathematics

Benefits for students	*f*
Understanding the process of mathematical knowledge formation	8
Enhancing their learning	6
Being encouraged to do research/science	5
Internalizing the idea that mathematics is a human creation	3
Gaining a rich perspective on mathematics	2
Benefits for teachers	*f*
Enhancing the content knowledge	8
Gaining prestige	5
Improving job satisfaction	2
Enhancing/strengthening communication with students	1
Benefits for teaching process	*f*
Supporting meaningful learning	18
Leading permanent understanding	3

The teachers pointed out the students may benefit from using the history of mathematics in the classroom as it provides opportunity to (a) understand the process of mathematical knowledge formation, (b) enhance their learning, (c) being encouraged to do research/science (d) internalize the idea that mathematics is a human creation, and (e) gain a rich perspective on mathematics. In the Table 8, the results were summarized with quotations from the teachers' views.

Table 8. The Students' Benefits from the History of Mathematics

Students' Benefits	*f*	*Middle School Mathematics Teachers' Views*
Understanding the process of mathematical knowledge formation	8	"*Children want to know how mathematics is discovered. They wonder where will it work, where did it come from*." T16
		"*It [the history of mathematics] could be used for explaining why we need mathematics, such as why it was discovered, what the initial driving forces were in the past*." T25
Enhancing their learning	6	*"… a few years ago, when I learned why mathematics emerged and its history, I realized that I better understood mathematics, so I think that the students could understand better in that way*." T7
		"*I think it's important to lay down the mathematical grounds of the topic*." T23
Being encouraged to do research/ science	5	"*The students sometimes wonder while I am teaching, 'How did they discover it teacher?', 'Can we discover something, too?', 'Is there anything discovered recently?' they ask. I say 'Yes you can*." T13
		"*As a result, when we investigate our own history, students can say: 'If people like me did it, I can do it' and be encouraged*." T18
Internalizing the idea that mathematics is a human creation	3	"*With the history of mathematics, it is possible to help students to figure out that mathematics is invented by people, not created in a vacuum, it is the product of human, and thus can be solved and done*." T11
Gaining a rich perspective on mathematics	2	"*To diversify emergence of numbers, to relate types of numbers… to make feel the numbers other than the all known classical natural numbers, whole numbers known to children, I mean they should see the existence of other numbers, they could grow the thought that these numbers are needed as well, they are used in math indeed*." T5

The middle school math teachers also pointed out that the use of the history of mathematics may help for teachers to "enhance their content knowledge", "gain prestige", "improve job satisfaction", and "enhance/strengthen communication with students". The results on the benefits of using the history of mathematics for teachers were summarized in Table 9.

Table 9. Teacher's Benefits from Using the History of Mathematics

Teachers' Benefits	*f*	*Middle School Mathematics Teachers' Views*
Enhancing the content knowledge	8	"*Although I am a mathematics teacher, I have not done much research on the history of mathematics. I never questioned where this or that came from. But when we include it in the lesson, we will inevitably need to get prepared before the lesson. If we are expert in the field of mathematics, memorizing the formulas is not enough; indeed we should learn its history*" T25
		"*As a teacher, knowing the history of your own lesson is an important thing. At least, s/he will be aware of what s/he is doing*." T7
Gaining prestige	5	"*…I believe that it makes the teacher more powerful to talk about the history of mathematics while teaching. Because the student might say 'Look! The teacher knows this, too'. It makes the teacher stronger*." T3
		"…The student considers his/her teacher as a well-equipped with cultural and academic issues, subject matter expert. In this sense, the use of history of mathematics would be nice." T22
Improving job satisfaction	*2*	*"We did not have the opportunity to practice, because the courses we took at the university were theoretical. Here, at least I have the opportunity to use some of the theoretical knowledge that I learned. Saying 'This is something discovered by this person', 'This is something discovered by that person' or showing their pictures, I also enjoy."* T4
Enhancing/ strengthening communication with students	*1*	*"It also helps to connect or touch with the student. Since we are working hard on numbers, for instance, we cannot do anything special to touch with student, yet the history of mathematics offers some ways."* T9

The mathematics teachers participated in the study stated that using the history of mathematics might also contribute to improve mathematics teaching in terms of meaningful learning and permanent understanding. In Table 10, the results were summarized with quotations from the teachers' views.

Table 10. The Benefits of Using the History of Mathematics for Teaching Process

Benefits for Teaching Process	*f*	*Middle School Mathematics Teachers' Views*
Supporting meaningful learning	18	"*If the student knows its history, learns what it is for, then s/he listens more willingly. The lesson will be more productive. It has a positive effect on my class*." T21
		"*…It draws attention, for example, when I teach, somewhere during the lesson, I say: 'While the Egyptians were building the pyramids, they used similarity.' or 'Thales did this and that' and so on. It draws their [students'] attention…*" T3
Leading permanent understanding	3	"*…For example, I taught Thales Theorem about similarity. The students immediately said 'Oh, this is a Thales question; this is something that Uncle Thales did'. I believe it is more long-lasting because they keep in mind in this way*." T4
		"*I've always questioned my children about why and where they came from, I use the proofs, to make learning more permanent*" T19

On the other hand, some of the teachers emphasized that the use of history of mathematics in learning and teaching process may turn into disadvantage due to both student-related and teacher-related issues. As given in Table 11, the student-related issues were expressed as "finding the history of mathematics as boring"; "not being interested in the history of mathematics because it is not a requirement of the mathematics course" and "failing to associate the history of mathematics with mathematics". The teacher-related issue was stated as "loss of prestige and self-confidence" due to the teachers' poor knowledge about the history of mathematics.

Table 11. The Restricting-issues about Using the History of Mathematics

Student-related issues	*f*	*Middle School Mathematics Teachers' Views*
Finding the history of mathematics as boring	6	"*Verbal things would be problem for a student who is used to work with numbers. S/he may not like it very much. Because some students just focus on the numbers while studying on math. The history of mathematics is not very interesting to them. This may be a negative thing for them*." T6
		"*Those who don't like history or social study lessons might be biased towards the history aspect of math*." T25
Not being interested in the history of mathematics	6	"…*Some students see this [the history of mathematics] as a waste of time because they are used to testing. Sometimes, I show videos about mathematicians, for example, I hear such voices as 'Teacher, let's solve tests', 'We have exam, will they be in the exam?' from students*" T27
		"*It may also seem like a waste of time for the students. They are totally focused on the exam; it may seem a bit of a waste of time for them*." T10
Failing to associate the mathematics with its history	1	"*Perhaps the history of mathematics will sound too abstract because they will not be able to associate it with mathematics much*." T24
Teacher-related issues	*f*	*Middle School Mathematics Teachers' Views*
Loss of prestige and confidence	*4*	*"You need to have a strong mastery of the subject matter. If you don't have a mastery of the history, you can say something wrong in the examples you give. If the student also reads in a book and claims that this knowledge is not like this but like that, then you may contradict with the student. Then you will lose his/her trust, as well as your prestige and self-confidence*." T5

The results related to the teachers' current use of the history of mathematics in their classrooms revealed that only three teachers stated that depending on the topic, they integrate adequately the history of mathematics in learning and teaching process; while the rest of the teachers stated that they rarely use the history of mathematics due to the reasons given in Table 12.

Table 12. The Reasons for the Rarely-use of the History of Mathematics

Reasons	*f*	*Middle School Mathematics Teachers' Views*
Lack of historical perspective in the curriculum	9	"*Because of the curriculum, I mean the history of math is not included in the curriculum. We follow the curriculum and thus, we do not have to explain or teach.*" T11
		"*The curriculum is lack of the history of math, so there's a comfort due to that. But if it was included in the curriculum, I would have to mention it; at least I would have to get prepared for, spend some more time on it. I think I'm taking advantage of not having it in the curriculum*." T22
Teachers' inadequate knowledge about the history of math	9	"*I couldn't include it [the history of math] in every topic because I don't know that whether the other topics are related to it frankly. I'm trying to include it in the topics that I know. I don't know, if I knew I would include it and maybe use it more effectively*." T3
		"*If I had mastery on it [the history of mathematics], I would certainly discuss a bit with the students at the beginning of the lesson. Actually, I do not have much knowledge about the history of mathematics and also I have not research on it much, though it was my fault*." T21
No room for the history of math in exams	7	"*The fact that there is no question about it [the history of math] in the exams affects a lot. A student thinks 'Such questions will not appear in exams, so it doesn't matter whether I learn it or not*." T24
		"…*The system is also built on exam-based approach. Everything is measured by tests or written exams. Among them, there is nothing about the history of math. So, for students, the number of correct answers in a text is important. The history of mathematics is not asked in exams.*" T15
Time constraint	5	"*I already have so much to cover in class, I have no time doing such activities as 'Wow, who found it', 'Where it came from' and so on. If we had more time, we could think how we could spend this time, maybe we could figure out the use of it [the history of math]. But we never have the chance to do that*." T20
No room for the history of math in the textbooks	5	"*I don't do anything particularly different from the textbook. The textbook approved by the Ministry of Education is not included in it, so I think that it is not necessary. In my opinion, if it is considered as essential, then it will be in the textbook.* " T8
Overloaded curriculum	2	"*There are a lot of content. The curriculum is overloaded, and due to the covering all topics in the curriculum, we have problems with the use of it*". T16
Students' inadequate desire to learn	2	"*I mention it [the history of math] only when the students ask 'What it is used for'… We explain when the student asks a question but if they ask nothing, we don't talk about it. We start the lesson directly*." T9

Lastly, the middle school mathematics teachers proposed the ways to effectively integrate the history of mathematics in the teaching and learning process. Their recommendations are presented in Table 13.

Table 13. The Teachers' Recommendations for Effective Use of the History of Mathematics

Teachers' Recommendations	*f*
Providing in-service training programs	20
Organizing student-centered learning environment	10
Inclusion of history of math in the curriculum	10
Preparing instructional materials and resources about history of math	8
Inclusion of history of math in textbooks	5
Reducing the intensity of the curriculum	3
Using the history of math in elective mathematics courses	3
Increasing the course hours	1
Inclusion of the history of math course in pre-service teacher education program	1

DISCUSSION AND CONCLUSION

The present study was carried out to investigate the faculty members' and the mathematics teachers' perspectives regarding use of the history of mathematics in the learning-teaching process of mathematics. The findings revealed that almost all of the middle school mathematics teachers and all of the faculty members believed that the use of the history of mathematics is necessary and important. The result is consistent with the findings from previous studies in the literature (Baki & Yıldız, 2010; Dejic & Mihajlovic, 2014; Hatisaru & Erbaş, 2012; Tokay, 2019; Yevdokimov, 2007).

In addition, both teachers and faculty members expressed that the use of the history of mathematics is a worthwhile effort, since the history of mathematics might help students to understand the process of mathematical knowledge formation, to enhance their learning, to gain rich perspective on mathematics. The previous studies in the literature support the findings of the study. For example, Ho (2008) argued that the history of mathematics helps students to understand the process of mathematical knowledge formation. Similarly, according to Liu (2003), students can better understand that mathematics is a constantly developing discipline with the use of the history of mathematics. Also, Siu (1993) stated that it can help students to gain a rich perspective of mathematics. Moreover, Tözlüyurt (2008) found out that mathematics lessons involving the activities selected from the history of mathematics can provide students to gain different perspective of mathematics. Similarly, the studies of Dickey (2001), Awasonya (2001), and Nataraj and Thomas (2009) also supported that the history of mathematics can support better understanding of mathematics. Besides, the study revealed that the use of history of mathematics may provide internalizing the idea that mathematics is a human creation for students. This result was also in compliance with Kaye's (2008) study. Hence, it can be concluded that the both teachers and faculty members appreciate the importance of the history of mathematics for offering more meaningful and enriched learning experiences to students. In other words, they seem to be aware of the potentials of integrating history of mathematics into learning and teaching process in designing well-qualified and rich mathematics learning environments. Moreover, the middle school math teachers and faculty members stated that the history of mathematics may help for teachers to enhance their content knowledge. Besides, the teachers expressed gaining prestige, improving job satisfaction and enhancing communication with students as the strengths of the use of history of mathematics, while the faculty members emphasized the student-centered instruction. In the literature, it is pointed out that teachers, while blending their qualified knowledge about the history of mathematics with in-class activities, can develop their creative skills and acknowledge the reason for teaching each specific topic. As a result, their teaching skills might improve (Furinghetti, 1997; Liu, 2003; Nataraj & Thomas, 2009; Pengelley, 2002). Furthermore, it is asserted in the literature that if the teachers are interested in the history of mathematics and integrate it in the learning-teaching process this may improve their knowledge, skills and attitudes about mathematics (Panasuk & Horton, 2013; Tymocsko, as cited by Liu, 2003). Similarly, in Ulusoy and Girit-Yıldız's study (2019) it is also found that the history of mathematics is necessary in order to enhance teachers' content knowledge and prestige. Besides, the middle school math teachers stated that the history of mathematics could support meaningful learning and lead permanent understanding and these results were supported with many research studies in the literature

(Fauvel & Maanen, 2002; Ness, as cited by Fried, 2001; Rickey, 1995). The previous studies also reported that the use of the history of mathematics in learning and teaching process could increase students' interest and curiosity towards mathematics and thus, it could support meaningful learning (Ersoy, 2015; Hatisaru & Erbaş, 2012; Tong, Loc, Uyen & Thi Y, 2019). In this respect, it can be said that the mathematics teachers who participated in the study were aware of the importance of the history of mathematics in the learning and teaching process of mathematics.

Besides the strengths, the participants mentioned some limitations about the use of the history of mathematics in classroom. The middle school mathematics teachers associated these limitations with both student- and teacher-related issues; while the faculty members linked them to only teacher-related issues. Both agreed on that teacher could lose their prestige and confidence if they have poor knowledge of the history of mathematics. In addition, some of the faculty members stated that if a teacher has poor knowledge on the history of mathematics, s/he has to deal with extra workload to comprehend and understand it. In this respect, it is concluded that teachers' poor/lack of knowledge was one of the reasons preventing teachers from integrating the history of mathematics in their class. The findings of the present study indicate similar patterns with the literature (Baki & Yıldız, 2016; Başıbüyük, & Şahin, 2019). Unlike the faculty members, the middle school math teachers stated that the student-related issues, namely finding the history of mathematics as boring, not being interested in the history of mathematics and failing to associate the history of mathematics with mathematics, also affected their decision-making process whether to use the history of mathematics or not. There are some studies that underline the similar student-related factors affecting the use of history of mathematics in teaching process. For instance, in the Siu's list (1998) of negative effects of the history of mathematics on students, it was stated as "…find the history no less boring than mathematics" and "lack of assessment" (as cited by Tzanakis & Arcavi, 2000, p. 203). Hence, the use of history of mathematics in the learning and teaching process might be affected by students' characteristics (e.g. beliefs, attitudes towards math, readiness level, etc.) as well as the teachers' characteristics (e.g. pedagogical content knowledge and skills, etc.).

Moreover, the majority of the teachers stated that they rarely used the history of mathematics in their classes. Lack of historical perspective in the curriculum, teachers' inadequate knowledge about the history of mathematics, no room for the history of mathematics in the exams, time constraint, no room for the history of mathematics in the textbooks, overloaded curriculum and students' inadequate desire to learn were the reasons of rarely-use of the history of mathematics in the learning-teaching process stated by the teachers. Besides, all of the faculty members expressed that the use of the history of mathematics in the classes was limited because of the teachers' inadequate knowledge about the history of mathematics, exam-oriented education, MoNE's uninterested approach, overloaded curriculum, lack of historical perspective in the curriculum and teachers' extra responsibilities were the factors affecting the use of the history of mathematics in learning-teaching process. In this respect, the present study clearly revealed that both the teachers and the faculty members perceived the history of mathematics as complementary part of mathematics teaching, but they also underlined such dynamics as education system, curriculum, instructional materials, time dedicated to math lessons, etc., that undoubtedly affect teachers' decision on whether to integrate the history of mathematics or not. The findings of the present study coincide with the related literature. For instance, Fried (2001, p.394) stated that "Teachers must complete a great number of topics in a very short time…it is not surprising that teachers should resist introducing a program of history of mathematics despite its virtues." Similarly, Siu (2007) pointed out that it might be difficult to use the history of mathematics in learning-teaching process if teachers have inadequate knowledge about it and also the history of mathematics does not affect the students' grade. In addition, Sözen's (2013) study indicated that teachers are challenged while using the history of mathematics in their classes because of having inadequate knowledge about the history of mathematics. In the study of Baki and Yıldız (2016), the most important factors that seem to prevent teachers from using the history of mathematics in their classes are the poor quality of the historical parts in the textbooks, inadequate numbers of course hours and the pressure of the central exam. Similarly, Panasuk and Horton (2012) found that teachers avoid using the history of mathematics due to lack of adequate knowledge on the history mathematics as well as not knowing how to use it in their classes.

Furthermore, providing in-service training programs, organizing student-centered learning environment, inclusion of history of mathematics in the curriculum, preparing instructional materials and resources about history of mathematics and inclusion of the history of mathematics in pre-service teacher education programs are the recommendations stated by both middle school mathematics teachers and faculty members. In addition, teachers proposed some other ways to effectively integrate the history of mathematics into learning and teaching process such as inclusion of the history of mathematics in textbooks, reducing the intensity of the curriculum, using the history of mathematics in elective mathematics courses and increasing the course hours; while faculty members emphasized that inviting more faculty members having deep knowledge about the history of mathematics to the curriculum development commissions for using the history of mathematics effectively in learning and teaching process. As a result, it can be concluded that organizing the pre-service and in-service teacher education and mathematics curriculum are priorities for integration of the history of mathematics into learning and teaching process. In other words, it is clear that both the teachers and the faculty members in the study give priority in their suggestions to increasing teachers' level of knowledge about history of mathematics and improving the aspects of the curriculum.

The present study is a phenomenological research focusing on the use of history of mathematics from the perspectives of the mathematics/mathematics education faculty members and the middle school mathematics teachers. Overall, as given the Figure 1, the findings of the current study clearly answered the following questions "Why the history of mathematics should be integrated in teaching and learning of mathematics?", "What is the current situation of the use of the history of mathematics in classrooms?", "What are the reasons for rare-use of the history of mathematics?" and "What should be done to use the history of mathematics effectively?".

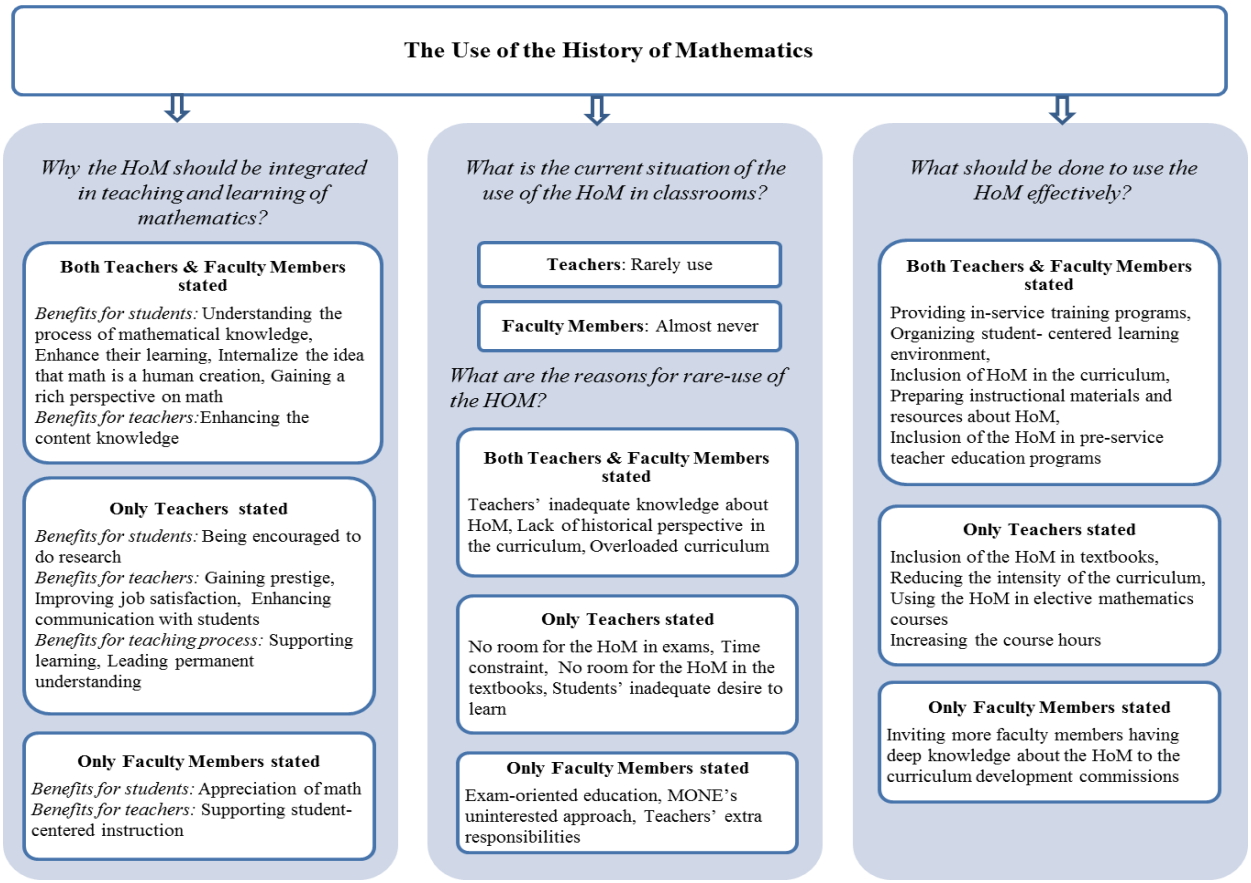

Figure 1. The history of mathematics from the perspectives of faculty members and teachers

According to the Figure 1, both the teachers and the faculty members who took part in the study stated that the history of mathematics should be used, yet the most-agreed reasons avoiding the use of history of mathematics are teachers' inadequate knowledge about history of mathematics. In this respect, the results

call for the improvements in teacher education programs (pre- and in-service) in order to equip teachers with the knowledge of the historical side of mathematics as well as how to integrate it in mathematics teaching. Moreover, the issues related to the current mathematics curriculum (e.g. no place for the history of mathematics, time constraint, etc.) should be solved. It is recommended that stakeholders (e.g. curriculum development experts, textbook authors, decision-makers, etc.) should improve both the curriculum and textbooks by considering the possible contributions of the history of mathematics for students, teachers and learning-teaching process pointed out by the teachers and faculty members in this study. Lastly, the results of the present study were only based on the views of the faculty members and mathematics teachers gathered from the qualitative data collection method. Thus, there is a need for additional research to get a wider picture about students' opinions and experiences of the history of mathematics in actual classroom practices.

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