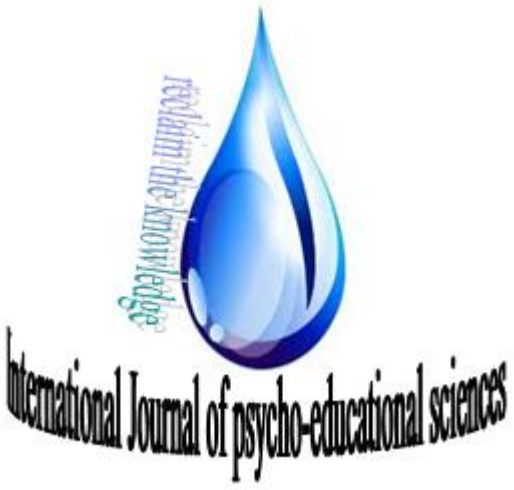

The Use of Addition/Subtraction Operation: Problem Equation Relationship Sample

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Abstract

This research was conducted to determine which method of operation is more commonly preferred (linear / lateral) in the process of students' learning addition and subtraction operation skills. A sample was created from the primary education 2nd grade students at various levels of success at mathematics lessons. The students' skills at performing these operations were examined using a success test consisting of 20 questions, along with semi-structured interviews with fifteen students. Results of this research show that students use the linear method for performing addition/subtraction operations. Their answers to the questions in the interview indicate that they prefer the lateral method when solving semi-abstract problems and those associated with the numerical axis. Students vacillate in determining the priority of each operation when solving the problem

Keywords: Addition/subtraction operations, lateral operations, linear operations, verbal problem solving.

Introduction

The need for effective mathematics skills, and their use in daily life, has increased. Mathematics facilitates one's life. It enables the individuals to analyze and explain their experiences and opinions, the behaviors of creative thinking and reasoning, patience, generating and using information and determination towards the events (Ministry of National Education (MEB), 2006). In addition, it helps the importance of mathematics education thanks to the need for mathematics in all fields of life.

There is a common belief that the mathematics lesson is the most difficult for students. In terms of learning the skills, as well as using them successfully, is learning mathematics different from other subjects? And is using a different learning method necessary in order for students to succeed at mathematics?

Mathematics has a different structure (abstract) compared to other subjects. It may not always be possible to bind all the information learned in the lesson or to express the information in a semi-abstract way. According to Bloom (1983), the presence of earlier learnings facilitates further learnings; however, the lack of prior learnings makes further learnings difficult. A mathematics lesson is a good example of this axiom, because there are prerequisites for learning (cited in Kelecioğlu, 1993). When students are unable to master certain mathematics subjects or one of the sub-parts of a subject, they most likely will encounter difficulties in establishing horizontal–vertical associations in the succeeding parts of the related subject or between the subjects or other lessons (Kelecioğlu, 1993; Swadener & Soedjadi, 1988).

Accelerating the development of analyzation skills, developing problem-solving skills and providing individuals with an advanced level of ability to reason are the aims of mathematics education (MEB, 2006; NCTM, 1989; NCTM, 2000). Developing problem-solving skills is a crucial factor in achieving these objectives. (Mestre,1991). In problem solving, Polya's analysis is commonly used: "1: understanding the problem; 2: determining the solution strategy and writing the mathematical sentence (equation) related to the problem; 3: solving the equation (or applying the strategy); 4: evaluation" (Altun, 2008). If a student is able to use these four steps when solving a problem on any subject (if he is able to express), it can be said that he has learned the related subject.

The problem-solving behaviours of primary school students were examined (Altun, 1995; Erden, 1986; İskenderoğlu, Altun and Olkun, 2004; Yazgan and Bintaş, 2005). Erden (1986) stated that 2nd grade students are successful at expressing a problem if they

can use their own words in writing the operations and the rules for solving the problem. There are also studies that examine the skills of primary school student skills in performing addition operations, and in which linear addition is discussed (Carpenter et al., 1999; Riley et al., 1983).

In literature, it is interesting that there are few studies on the expression of verbal problems as a mathematical sentence (equation) or writing a mathematical sentence in the form of an verbal sentence (problem).There are also studies asserting that students have difficulty in transforming verbally expressed problems into into the form of an equation (Dede, 2004; Lodholz, 1990; Mayer, 1982). Based on these studies, it can be said that students cannot transpose a problem which is expressed verbally, into the form of an equation. Being able to convey a verbal expression (problem) as an equation or a verbal expression (problem) which is appropriate for the mathematical expression is quite difficult for students of all age groups.

Being able to create a verbal problem and express it as a mathematical sentence - are included in the curriculum from the 1st grade of primary education. Unless students learn how to express verbal problems as equations, these skills are extremely difficult to develop as their education progresses (Kieran, 1992; Silver, 2000; Pawley, Ayres, Cooper & Sweller, 2000). For this reason, students' skill levels (for creating verbal problems) should be tested from the 1st grade of primary education and onwards.

According to Rosnick (1981), the difficulties encountered by students during this process are due to the fact that the sub-parts of a subject are not understood completely. This is not a well-known point and often insufficient effort is made to compensate. This skill deficiency in students can be associated with the sub-learning parts of writing a mathematical sentence. The sub-parts of writing a mathematical sentence include performing operations using lateral, linear or various other methods (such as counting, analyzing, etc.). Especially while providing students with basic operation skills, verbal problems should be used first and the expression of a problem and the operation performed should be consistent. For example; "I had 3 balls. My brother gave me 2 more. How many balls do I have now?" While solving this problem the symbol "+" should be expressed as "gave"; and the symbol "=" should be expressed as "now". It is not effective to rush the process of expressing the names of the symbols because students do not know how to read and write (Albayrak, 2000). The present study was carried out mainly in three subtitles.

1. In the process of providing students with the operation (addition/subtraction) skills: establishing a relationship between the concept and the operation,
2. Expression of verbal problems mathematically,
3. Performing the operations while solving problems in different ways.

The present research can also be associated with the addition/subtraction operation, one of the sub-parts of writing a mathematical sentence, which is the second phase of Polya's problem-solving strategy.

Method

Data Collection

A case study was carried out within the present study, using primary school 2nd grade students. Students were chosen randomly from the 2nd grade classes of the two primary schools. The group consisted of 15 students from each school (in total 30) with above-average skill level in mathematics lessons; 20 students (in total 40) of average skill level; and 15 students (in total 30) of below-average skill level. 5 students (average skill level) was excluded from the study for behavioral issues. In total, 95 students participated

in the study. The students were given a test consisting of 20 questions, divided equally between lateral addition, linear addition, linear subtraction, lateral subtraction and verbal problems. Addition operation questions included the following:

Question 1: the addition of two one-digit numbers
Question 2: the addition of one two-digit number and two one-digit numbers
Question 3: the addition of two two-digit numbers and one one-digit number
Question 4: the addition of three two-digit numbers

Subtraction operation questions included the following:
Question 1: the difference between two one-digit numbers
Question 2: subtracting a one-digit number from a one-digit number
Questions 3 and 4: subtracting a two-digit number from another two-digit number.

The verbal problems used are of combined and piece- piece total according to the classification made by Carpenter and his colleagues (1999). All four problems question the unknown total and the unknown result. A semi-structured interview was also made in line with the students' answers.

There were two questions included in the interview to assess their problem solving skills. One of these questions tested the skills used in performing an operation using semi-abstract; and the other tested problem-solving skills using the numerical. In addition, the addition/subtraction operation, and the difficulties encountered while solving the verbal problems related to these operations, were discussed in the interview. In the interview, students were asked which operation method took first priority in the lessons (lateral or linear). Some of the interviews with the students are outlined in the findings section. Students' real names are not used in this study; they were assigned pseudonyms by the researcher.

The interview questions were prepared according to the answers of the students in the success test. Our aim in the interview was to determine where students have difficulty and which operations take top priority in lessons. Experts, course books and related literature were consulted in preparing the interview questions and success test. Experts consisted of three academicians who work in the department of primary school mathematics education and two classroom teachers who work within the body of the Ministry of National Education. While applying the success test to the students, the students were not guided, ie. "correct, wrong, you should revise the solution, are you sure that the answer is this..." To conduct a proper measurement the questions of the success test were not written as a whole, but were previously written as a single problem in such a way to be sufficient for the students. After one question was answered completely, the next question was given to students. In this way, all twenty questions were given to the students. The length of time to solve each problem was measured with a chronometer. The length of time to solve verbal problems was not measured.

Data Analysis

In the present study, 1 (one) point was given for the correct answers; 0 (zero) point was given for incorrect answers or unanswered questions. The Kuder-Richardson Formula 20 (KR-20) was used to measure the reliability of this study (Atılgan, Kan and Doğan, 2009; Tekin, 1993). The questions in the success test in this study were given to 60 students before the actual application. The KR-20 internal consistency coefficient was found to be 0.76.

Results and Interpretation

The number of seconds to perform the lateral and linear addition/ subtraction operations for the students within the scope of the present study is given in Table 1.

Table 1: *Students: length of time to perform operations*

Question orders	Linear addition/ subtraction questions solve times (average /in seconds)	Lateral addition/subtraction questions solve times (average/in seconds)
1.	5	7
2.	10	13
3.	11	15
4.	11	16

According to the results in Table 1, students spend more time solving lateral addition/subtraction operation questions compared to the time spent solving linear addition/subtraction questions. These results are the same for lateral and linear subtraction operations. The results indicate that students use linear operations more often.

The data in Table 2 indicates that students perform linear operations most successfully; their performance decreases when using lateral operations and solving verbal problems. These results can be considered as an indicator that the students use linear operations more commonly in lessons. Success levels may decrease in problem solving because sufficient time is not allocated for teaching concepts (when to perform which operation).

Table 2: *Correct answers: statistics*

Students	Linear addition/ subtraction		Lateral addition/subtraction		Verbal problems	
	f	%	f	%	f	%
Above-average class level	30	100%	25	83%	20	66%
Average class level	30	86%	20	66%	15	43%
Below-average class level	20	66%	15	45%	5	17%

The answers of one student from each student group are given below. The objective is to provide information about the interviews with students. An interview with student Ahmet, of above-average class level, follows:

Researcher: *Ahmet, when I analyzed your solutions, I understood that you perform linear (addition – subtraction) operations in a shorter time. Which method do you use more in your class, linear (addition/ subtraction) operations or lateral (addition /subtraction) operations?*

Ahmet: *We perform linear (addition/ subtraction) operations most. This is because it is easier to perform operations in this way.*

Researcher*: Well Ahmet, which question that we asked you was the most difficult one for you?*

Ahmet: *None of them was difficult, I solved all of them.*

Ahmet's answers in the interview are consistent with his answers in the success test. Ahmet answered all the questions correctly. However, he spent a little more time on the lateral (addition/ subtraction) operations and on the verbal problems. The answers given by other above-average class level students were similar to those of Ahmet.

The answers of the average class-level students can be summarized as follows. Some of the students (five students) in this group worked on the lateral (addition/ subtraction) operations, by rewriting them in a linear form. Our teacher gives us linear (addition/ subtraction) operation more frequently in lessons. We are rarely given (addition/ subtraction) operations. As we do not do them as frequently, it takes more time to solve them. Therefore we found it easier to perform operations in a linear way. Students stated that they found it difficult to solve the verbal problems. An interview with Student Pinar, who rewrote her questions in this way, follows:

Researcher: *…Well Pınar, which question that we asked you was the most difficult one for you?*

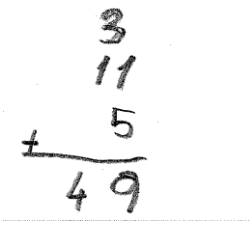
Pınar*: lateral (addition/ subtraction) operations.*

Researcher*:…You performed the (addition/ subtraction) operations that we asked you to perform laterally by writing them in a linear way. Why did you do this?*

Pınar*: Because it is easier, that's why I did in that way.*

Twenty students did not pay attention to the phase concept when writing the numbers in a linear way in solving the verbal problems. The operation used by one of these students is as follows:

Example: Alican is reading a story book. In the first day, he reads 11 pages of this book, in the second day he reads 5 pages and in the third day he reads 13 pages. How many pages in total did Alican read in this book?



Results of the study showed that the students of below-average class level are deficient in the skills used to perform the lateral addition/ subtraction operations and solve verbal problems. Fifteen of the below-average students were unable to even perform linear addition operations. They could not write the result in the lateral addition operations. The findings related to subtraction operation are the same. Again, these three students made a mistake carrying the decimal value to the next digit in the linear addition operation.

Moreover, it was understood that some of the students in this group wanted to answer the questions related to lateral addition operations by writing them in a linear way. Some students added the units digit of the first number to the two-digit of the second number when performing the addition of one two-digit number and one one-digit number operation. This situation is seen in the example above.

This may indicate an inadequate amount of instruction time. Students who did make an effort to answer the verbal problems used only a part of the numbers used in the problem while performing the operations. A small proportion of the students used a multiplication operation while they should have used an addition operation. The interview with Student Adem, of below-average class level, follows:

Researcher: *Adem, you solved the problems related to lateral addition operation by writing them in a linear way. Why did you do this?*

Adem*: It is easier to do so...*

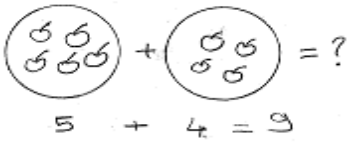
Researcher: *Well Adem, which method do you use more in your class, linear (addition/ subtraction) operations or lateral (addition/subtraction) operations?*
Adem*: linear*
Researcher*: which questions were you unable to solve, of the questions that I gave you?*
Adem: *well, these (he is showing the verbal problems)*

From the answers given by the students to the interview questions, it was found that, in class, they use linear addition/ subtraction operations most often..

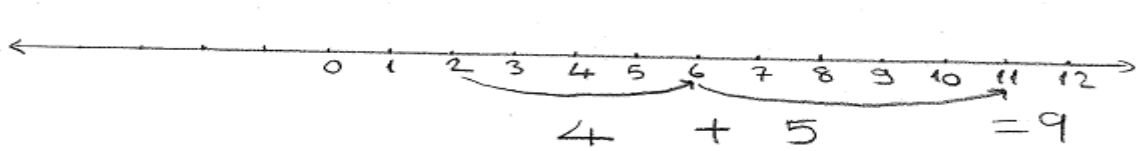
The results indicate that the majority of the students' skill-sets are inadequate for solving the verbal problems. Approximately 60% of the students are lacking adequate lateral (addition/ subtraction) operation skills. These students' answers are consistent with the answers that they gave to the questions in the success test and in the interview. All the students solved the verbal problems performing linear addition operations.

The majority of the students used the lateral addition operation to solve two of the questions asked in the interview (not included in the success test). These questions, and the students' answers, are as follows:

What is the result of the addition operation which is shown in the figure below?

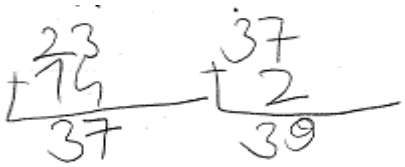


Question is; "Perform the addition operation given in the numerical axis below".

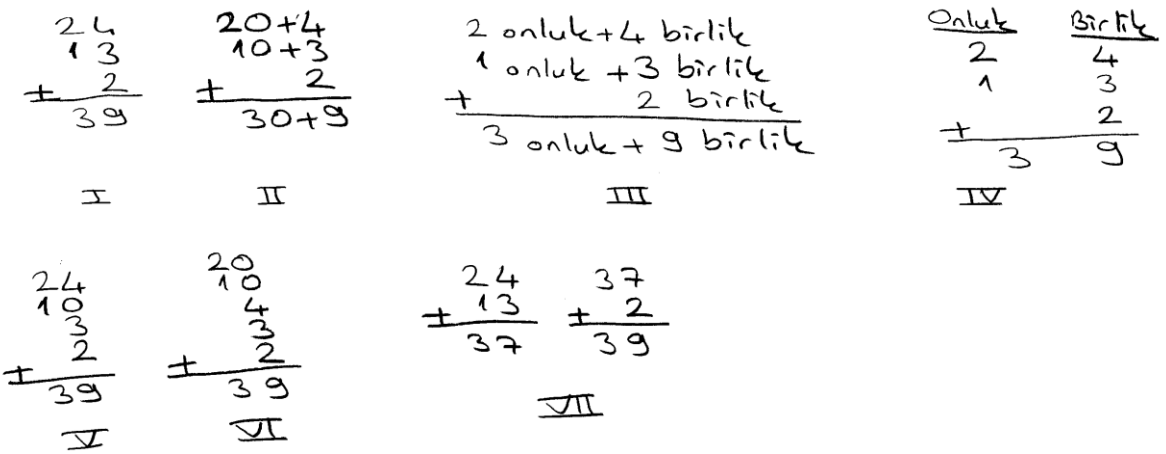


Results of analyzing the solutions of the verbal problems show that the students did not try a different method of solving them. Only two students performed an addition operation with three numbers by dividing them into 2 groups. Below is an example:

Example: Ali, his mother and his father go to pick tea and they weigh the teas that they picked at the end of the day. Ali's father picked 23 kg tea, his mother picked 14 kg tea and Ali picked 2 kg tea. In total, how many kilos of tea did they collect at the end of the day?



There may be a few different solutions of the above question: (these are examples authors' suggestions)



When students are taught only one method of problem-solving that is what they will use. When they are taught more than one method, the scope and quality of their insights is increased and broadened.

Evaluating students along these lines, it was seen that only ten (1st and 7th) of the solution types listed above were used. Only ten of the students used the 7th solution method. Why the majority of the students used the 1st solution method can be explained: they do not know other solution methods.

Using a variety of problem-solving strategies to perform operations can contribute greatly to the development of students' skill sets (Albayrak, 2000, pp. 111-114). In addition, the mistakes made by the students (writing the numbers in a linear way) can be eliminated using the 2nd, 3rd and the 4th solution ways above.

The results of the student interviews agree with those of the success test: "linear (addition/ subtraction) operations are the ones most commonly used in the classroom environment".

Discussion

It is appropriate to begin with problems when teaching basic operations to the students. According to the results of the study, it is impossible to say that this approach is adopted. The difference between the accomplishments of the students in performing operations and those in problem solving is great. That the students vacillate when choosing which operation to apply to which problem confirms this.

Teaching only one way of solving a problem ensures that students will use that method, but at the cost of using other, perhaps more effective methods.. That the operations which were asked to be performed laterally were performed in a linear way by the students confirms this idea.

It is frequently necessary to use concretization, semi-abstract or numerical axis when solving problems using the basic operations. In such problem solving, it can be said that lateral and linear operation methods are used at the same rate. However, it is indicated that lateral operations use decreased when the students started performing operations with two-digit numbers and they do not use this operation method after a while.

The use of linear and lateral operations in solving verbal problems should be given equal time at the same rate from the first grade of primary education. The students should be encouraged to perform lateral operations by using expressions like "linear operations cover a lot of ground, from now on let us use the lateral method". This is because lateral operations may be preferable to writing mathematical sentences in the following grades.

Some of the students performed the operation in the success test incorrectly. Performing the operations correctly is undoubtedly important. An effective way to facilitate this is to associate the operations with counting. Associating the basic operations with counting (counting forward one-by- one starting from one number as many as the other number, addition; counting backward one by one starting from one number as many as the other number, subtraction; multiplying the same numbers instead of adding; dividing a number instead of subtracting the same number from it again and again) can be easily performed (Albayrak, 2000). The performance of operations will be made attractive and easy thanks to associating counting with operations.

Given the relationship between counting and mental development, it is known that the students who cannot count can learn how to count in time. Therefore, it would seem that this skill can be taught later. However, the inability to solve problems turns into fear if these deficiencies in problem-solving are not addressed immediately. It is difficult for students to overcome this fear.

While a significant number of students are successful at performing operations, they fail in problem solving. One reason why the majority of the students failed in problem-solving is because they do not know which operation to use. Which operation to be used first in solving a problem or, in other words, the order of the operations is understood from the analysis of the problem (Albayrak, İpek, Işık, 2006). While discussing the methods of solving a problem, expressions like "Did it increase?", "Did it decrease?", "As it increased, it is addition; as it decreased, it is subtraction..." should not be used. While providing the students with this kind of skill, the concept of the operation in the sample problems should be emphasized. This is because the students who are able to perceive the order of the operations to be performed can also solve the problems. In order for students to understand which operation to perform first, it will be useful to solve two or three sample problems. Using these applications on different problems may eliminate the issue. Hence, it is useful to follow the order of concept and operation when solving a problem. In addition, time should be allocated for different counting methods, which can be followed in performing operations (counting in tens, performing operations on the same types after analyzing, etc.). Therefore, knowing how to solve a problem in more than just one way is of great importance for performing operations and thinking, as well.

Lateral operations and mathematical symbols are necessary for writing a mathematical sentence (equation) which may correspond to oral problems. Using the problems which include the addition and subtraction operations in which there is a variable or unknown is an appropriate approach when beginning to learn how to write mathematical sentences. Moreover, the expression "$5 + \blacksquare = 8$" should be used in addition to the subtraction operation for a problem like "I had 5 balls. How many balls does my brother need to give me so that I have 8 balls?" Therefore, abstract thinking and being able to express thoughts using symbols become more meaningful.

Conclusion and Suggestions

The students in the sample group took longer to perform lateral (addition/ subtraction) operations than linear (addition/ subtraction) operations. Respondents made more mistakes in lateral operations because they are not as accustomed to using them. Being unable to solve the problem while answering the oral problems arose from the deficiencies based on the concept. Students who were able to determine which operation should be used were able to solve the problem without any mistakes. The students who used lateral operations are included in this group.

There is a parallel between the findings of the present study and the findings of the similar previous studies on this subject (Altun, 1995; Carpenter et al., 1999; Erden 1986;

Riley et al., 1983). In the present study, the use of addition/ subtraction skills of the students in the process of problem solving was examined. A new study may be conducted that also includes multiplication/division operations.

Sentence writing is a skill required for mathematical thinking and expressing what you think mathematically. Performing operations and expressing these operations as symbols are a universal expression of thoughts. The association between the concept and the operation should be established from the first year of primary education. The subject should be expanded to include teaching other ways to perform the operation. Lateral and linear operations should be substituted. Teachers should gradually work up to symbol use. Teaching students to express a verbal problem as an equation and to create a verbal problem that corresponds to the expression given as equation should be included in the curriculum.

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