The Impact of Dialogic Teaching on Academic Success and Anxiety Regarding Mathematics Courses

Abstract: The In this study, the aim is to examine the impact of Dialogic Teaching on students' academic success and anxiety regarding mathematics subjects of limit and continuity, which are in the scope of 12th grade mathematics curriculum, within the sub learning domain of continuity. During the research, both qualitative and quantitative methods were employed. The sample comprises of 56 students, 27 of which were the experimental group and the other 29 were the control group. Data sources consist of a continuity sub-learning domain success scale, which was developed by the researchers; a mathematical anxiety evaluation scale, which was revised with concept cartoons; and video recording of the lectures. During the study, Dialogic Teaching was used in the experimental group, while curriculum was taught in the control group. The results of the study indicate that Dialogic Teaching was not only effective in increasing students' success in the sub learning domain of continuity, but also helpful in reducing mathematical anxiety among students. The drawn conclusion was that Dialogic teaching has improved students' ability to generate alternative solutions to a problem, form and justify theses, make evidence-based judgments. Also it was effective in enabling students to comprehend concepts more profoundly by making scientific decisions.

Keywords: Curriculum, instruction, dialogic teaching, mathematical education

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Introduction

As a product of certain thinking processes, the mathematical information, due to its nature, should be scientifically founded and should not loose its strength before its anti-thesis (Skirbekk and Gilje, 1971; Bakhtin, 2010). In this context, based on the necessity that theses should be scientifically founded, in-class applications in mathematical education should conform to the nature of mathematics and the creation stages of information (Bingölbali, Arslan and Zembat, 2016). In order to realize this, it should be ensured that during in-class applications, students can think like a scientist on problem situations and scenarios, which are structured in accordance with the curriculum, using mathematical thinking systematic and they use their skills of prediction, making assertions, justification, discussion and discussion-based problem solving, generalization and testing by making assumptions (Bingölbali et. al., 2016; Ministry of National Education of Turkey, [MEB] 2019).

It is emphasized that in mathematics course curricula, there needs to be mutually complementary relationships between computational and conceptual information types (Ministry of National Education of Turkey, [MEB] 2019). In mathematical education, conceptual and computational information complement each other and conceptual information is essential for application of computational information (Baki and Kartal, 2004; Soylu and Aydın, 2006). It can be said that the necessary conceptual information and mathematical concepts have a cumulative structure. Considering this cumulative structure, one of the most important deficiencies in mathematical education is to add new information and carry out calculations without thoroughly learning the underlying concepts (Soylu and Aydın, 2006). Doing calculations on a concept without learning it beforehand (Baki and Kartal, 2004) and moving onto learning another concept, causes low-quality learning since conceptual and computational information cannot complement one another. In order to increase learning quality, while teaching the concepts about a subject, certain methods, which enable revealing scientific arguments regarding these concepts and justification of those arguments' validity (NCTM, 1989; Ministry of National Education of Turkey, [MEB] 2019). In this context, a particular approach, which makes it possible to scientifically found and question the proposed arguments, should be used (NCTM 2000; Alexander, 2008). It can be said that Dialogic Teaching, which conforms to this approach, maps well with the nature of mathematics and the creation stages of mathematical information (NCTM, 1989; Bakhtin, 2010; Şahin, 2016).

Dialogic Teaching is a collaborative decision-making process, during which the students justify their arguments and lay a foundation for them (Toulmin, 1958; Douek, 1998; 1999). Within this process, the queries and their answers for the justification of an argument may harbor new questions in themselves. By answering these questions, the justification of the argument is complete. The comparison between justified arguments enable exchanging ideas within peers. In this sense, dialogic teaching can briefly be defined as the process of revealing ideas and it has 5 stages (Vygotsky, 1978; Wertsch, 1985; Juzwik, Nystrand, Kelly and Sherry, 2008; Bakhtin, 2010). In the scope of Dialogic Teaching, especially when making an introduction to a new subject, offering solutions to problems that are within the course-aligned scenarios, revealing solution-oriented arguments and justification of these arguments are quite effective while teaching a new concept or multiple concepts, which have preconditional relationships. Discovering numerous arguments, summary, comparison, justification and collaborative decision-making are the five stages of Dialogic Teaching (Toulmin, 1958; Vygotsky, 1978; Wertsch, 1985; Alexander, 2008; Juzwik et al., 2008; Bakhtin, 2010). It is possible to create teaching media that conform to the five stages of Dialogic Teaching during education of mathematical concepts.

Dialogic Teaching, which is being applied to different subjects, encourages students to think and take responsibility of their own learning (ouek, 1998; 1999; Alexander, 2008). By applying Dialogic Teaching's conversational strategies such as listening, asking for opinion, asking for explanation, asking for example/evidence, diversification and reformulation of ideas, a teacher can inspire student to form arguments about a subject or concept, create justifications for these arguments and as a result make sense of that subject or concept (Vygotsky, 1978; Wertsch,1985; Alexander, 2008; Bakhtin, 2010). In addition to student-teacher interaction, conversational strategies also increase the interaction among students (Juzwik et al., 2008). The initial step in the applications of Dialogic Teaching is to reveal as many different arguments as possible that students can muster via a question about a certain subject or concept. The teacher queries the students about new ideas about these arguments, then performs investigations, which enable diversification of these ideas, and requests explanations (Kuhn, 1994; 1995). In accordance with given explanations, the students are asked to pass evidence-based judgments. The arguments are compared according to their justifications and students are enabled to reach a collaborative conclusion (Vygotsky, 1978; Wertsch,1985). It is of paramount importance that the teacher provides guidance in order to fuse the ideas together during this process. Dialogic Teaching directs students to create arguments about the subject or concept at hand, justify or, if necessary, refute those arguments and examine the validity and reliability of the acquired evidence. As a result of this, students advance to reaching a conclusion. As a consequence of Dialogic Teaching application, decision, on which students agree, can be reached (Douek, 1998; 1999; Bakhtin, 2010).

The increasing complexity of cognitive skills and thinking processes causes a feeling of helplessness and worry about mathematical learning process. And this, in turn, creates anxiety of failing mathematics courses (Richardson and Suinn, 1972; Tobias and Weissbrod, 1980). Anxiety towards mathematics courses, which can be defined as feeling helpless against mathematical operations, and experiencing worry and mental derangement (Tobias and Weissbroad, 1980), prevents desired level of success and development in the field of mathematics. Studies about mathematical education and mathematical anxiety indicate that high level of anxiety about mathematics courses impacts success (Richardson and Suinn, 1972; Betz, 1978; Thomas and Higbee, 1999) and learning processes (Rounds and Hendel, 1980; Tobias and Weissbrod, 1980; McLeod, 1988; Vinson, 2001; Sloan, Daane and Geisen, 2002; Kurbanoğlu and Takunyacı, 2012) in a negative manner. The fact that students' high levels of mathematical anxiety and their consequent low academic successes, emphasizes the importance of researches conducted in this subject.

When the compatibility of Dialogic Teaching applications to the nature of mathematics and to the process of mathematical information generation is considered, it can be said that Dialogic Teaching applications can be utilized to increase success and reduce anxiety (Richardson and Suinn 1972; Betz 1978; Tall and Vinner, 1981; NCTM, 1989; Soylu and Aydın, 2006; Bakhtin, 2010; Kutluca, 2010; Şahin, 2016). By allowing students to freely express ideas and for justifications, Dialogic Teaching applications offer a significant increase in student success (Applebee, Langer, Nystrand and Gamoran, 2003; Applebee et al., 2003; Şahin, 2016). Several facts such as the limited number of studies regarding in-class applications of Dialogic Teaching and curriculum subjects of 12th grade (Yalçınkaya and Özkan, 2012; Güneş, 2013), the most difficult subjects to learn being limit and continuity (Tall and Vinner, 1981; Baki and Kartal, 2004; Akbulut and Işık 2005; Soylu and Aydın, 2006; Özmantar, and Yeşildere, 2008), students having numerous misunderstandings about these concepts and obtaining correct results about concepts by drawing wrong justifications (Aydın and Kutluca, 2010) augment the importance of this study and its contributions to the literature. Within this context, the aim of this paper is to examine the impact of Dialogic Teaching on students' academic success and anxiety regarding mathematics subjects of limit and continuity, which are in the scope of 12th grade mathematics curriculum, within the sub learning domain of continuity.

In the experimental part of this study, the following hypotheses were tested (H_0 : null hypothesis, H_1 : experimental hypothesis):

$H_0$1: There is no statistically significant difference between the anxiety pre-test scores of the students in the experimental group and control group.

$H_0$2: There is no statistically significant difference between the academic success pretest scores of the students in the experimental group and the control group.

$H_1$3: There is a statistically significant difference between the anxiety pre-test and post-test scores of students in the experimental group.

$H_1$4: There is a statistically significant difference between the academic success pre-test and post-test scores of students in the experimental group.

$H_0$5: There is no statistically significant difference between the anxiety pre-test and post-test scores of the students in the control group.

$H_0$6: There is no statistically significant difference between the academic success pre-test and post-test scores of the students in the control group.

$H_1$7: There is a statistically significant difference between the anxiety post-test scores of the students in the experimental group and the control group.

$H_1$8: There is a statistically significant difference between the academic success post-test scores of the students in the experimental group and the control group.

METHODOLOGY

RESEARCH MODEL

Both qualitative and quantitative research methods were employed concurrently in the study. A pretest/post-test quasi-experimental research pattern with a control group was used. This model is used to test the cause and effect relationship between the variables, which are controlled by the researcher and helps assessing the significance of difference between the pretest and the post-test (Cresswell, 2016). In the scope of the study, in addition to the qualitative data, quantitative data (video recordings and concept cartoons) were also obtained in order to examine the impact of interference (application of Dialogic Teaching) alongside with quasi-experimental pattern.

STUDY GROUP

The study group of the research consists of 12th grade students of an Anatolian High School in Antalya, Turkey. Since the subjects of limit and continuity are concepts handled at the 12th grade level, 12th grade students were studied within the scope of the study. Students' sections were assigned to experiment and control groups in an unbiased manner. There were a total of 56 students in the sample spaces, 29 (52%) of whom were female and the remaining 27 (48%) were male. The experiment group consisted of 27 students, 14 of whom were female (52%) and 13 were male (48%), whereas the control group consisted of 29 students, 15 of whom were female (52%) and 14 were male (48%). It is evident that both experiment and control groups have similar distributions regarding gender.

DATA COLLECTION TOOLS

In this study, Continuity Sub-Learning Domain Achievement Test (CSLDAT), which was developed by the researchers and used for assessing skill level in continuity, a sub-domain of limit and continuity subjects within 12th grade mathematics course, and Revised Mathematics Anxiety Rating Scale (RMARS), which was developed by Plake and Parker (1982) and adopted to Turkish culture by Akın, Kurbanoğlu and Takunyacı (2012) and utilized for measuring the mathematics anxiety of students.

CSLDAT was developed in order to determine the success of students in the continuity sub-learning domain regarding the five critical gains. Firstly, the gains of continuity sub-learning domain within Ministry of National Education's mathematics normalcurriculum in secondary education were determined. In order to ensure the research's content validity, a table of specifications was prepared. This table of specifications consists of gains in this sub-learning domain and the cognitive level, in which these gains will be measured. Cognitive levels were constructed in alignment with Bloom taxonomy. Questions were prepared according to the relationship between the gains and the cognitive domain. A target content relation was formed by indicating which target belonged to which subject. 8 questions were prepared by the researchers for each gain. A total of 40 multiple-choice questions consisted the item pool of the study. The questions in the pool and the table of specifications were examined regarding content validity from the perspectives of assessment and evaluation, program development and mathematical education by 3 experts per field, each of which had at least Ph. D.s in their respective field. The questions and the table of specifications were revised according to the feedback and recommendations given by the experts. Then, in order to determine the comprehensibility of the questions (with respect to clarity, simplicity and wording), a pretest form, which consisted of 40 questions, were fully applied to 30 12th grade students and their feedback were obtained. After analyzing the gathered data, the final questionnaire that included 20 questions that have the best distinctiveness and have mid-level difficulty while considering the distribution of the questions with respect to the gains. The KR20 reliability coefficient for the trial application was found to be 0.92.

The Revised Mathematics Anxiety Rating Scale (RMARS) is a 5-point Likert scale, which consists of two sub-scales such as mathematics learning anxiety scale (16 items) and mathematics evaluation anxiety scale (8 items). The points of the scale range between Never (1) and Always (5). In the adaptation study, the Cronbach alpha internal coefficients for the whole scale, the mathematics learning anxiety and the mathematics evaluation anxiety sub-scales were found to be 0.93, 0.91 and 0.88 respectively; and the corrected total item correlation was observed in the range of 0.30-0.80. As a result of Confirmatory Factor Analysis, Chi-square was found to be 533.57 (N=372, sd=242, p=0.00), RMSEA was 0.057, NFI was 0.96, CFI was 0.98, IFI was 0.98, RFI was 0.96 and SRMR was 0.053 (Akın et al., 2010).

In order to be able to examine the impact of applied interference in greater detail, to reveal in-class statements about dialogic teaching and to determine student interactions, the lectures were documented via video recordings in addition to the utilized scales. Furthermore, a concept cartoon, which was developed by Özbek and Uyumaz (2017), was employed in order to determine the student's level of comprehension regarding three fundamental conditions of continuity, and their missing and faulty information about the subject. Student responses about the concept cartoon were graded according to a rubric, an example of which is presented in Table 1.

Table 1. Concept Cartoon Example Grading

Cartoon Character	Responses of Student S5	Grading
1	I don't agree with the student in the cartoon. The graph is not continuous, which means the function is not defined at that point. It is not continuous.	Correct Remark – 1 point Justification Exists – 1 point
2	...agreed.	Correct Remark – 1 point Justification Does Not Exist – 0 point
3	...agreed. Presented the situation as a graph. In my opinion, this function is not continuous since it does not have a limit at that point.	Correct Remark – 1 point Justification Exists – 1 point
4	I definitely agree with the student. Another condition for continuity is to have equal right and left limits at that point. Since limits are not equal, it is not continuous.	Correct Remark – 1 point Justification Exists – 1 point
		Total Score: 7

DATA COLLECTION PROCESS

The syllabuses and in-class activities for both the experiment and the control group were designed by the researchers. The experiment group's syllabus and in-class activities were designed in alignment with the stages of dialogic teaching. While preparing the experiment group's syllabus, the common 5 stages of dialogic teaching (problem introduction, argument discovery, argument summary, argument comparison and decision-making) were followed while considering the gains in Ministry of National Education's lecture book and syllabus. Activities (activity-1 and activity-2), which contain scenario situations about concepts, and questions were designed by taking advantage of similar studies in the literature (Alexander, 2008; Juzwik et. al., 2008; Şahin, 2016) and conforming to fundamental sources (Ministry of National Education of Turkey, [MEB] 2019) and based on the necessity of asking questions and guiding in-class activities in dialogic teaching according to the nature of information. The scenario in Activity-1 was prepared for demonstrating the first two stages of dialogic teaching such as problem introduction and argument discovery. Whereas, the scenario in Activity-2 was prepared towards the argument summary and argument comparison stages of dialogic teaching. In the study, teaching application and data collection were completed after a 6-week time frame. The processes, which were carried out on experiment and control group within the scope of the study, were summarized in Table 2.

Table 2. Data Collection Process

Week	Experiment Group	Control Group
1	- Pre-test Application (CSLDAT and RMARS)	- Pre-test Application (CSLDAT and RMARS)
2	- Application of syllabus based on dialogic teaching - Activity-1 application (problem introduction and argument discovery) - Video recording of the lecture	- Application of syllabus and activities based on the teaching program - Video recording of the lecture
3	- Application of syllabus based on dialogic teaching - Activity-2 application (argument summary and argument comparison) - Video recording of the lecture	- Application of syllabus and activities based on the teaching program - Video recording of the lecture
4	- Application of syllabus based on dialogic teaching - Concept Cartoon - Exercise Pages - Video recording of the lecture	- Application of syllabus and activities based on the teaching program - Concept Cartoon - Exercise Pages - Video recording of the lecture
5	- Application of syllabus based on dialogic teaching - Practice - Video recording of the lecture	- Application of syllabus and activities based on the teaching program - Practice - Video recording of the lecture
6	- Post-test Application (CSLDAT and RMARS)	- Post-test Application (CSLDAT and RMARS)

As shown in Table-2, CSLDAT and RMARS pre-tests were applied to both experiment and control groups. In the scope of the study, after dialogic teaching application (interference) to the experiment group, and the application of the teaching program to the control group, CSLDAT and RMARS post-tests were conducted concurrently.

The lectures were applied by the main writer of this study in both the experiment and the control groups. The researcher tried to assume a role, who enables discovery, self-doing and problem-solving rather than a role, who merely transmits information, discovers it and does the work itself in both study groups. In order to increase students' in-class participation, same amount of hinting, relevant feedback and reinforcers were tried to be utilized in both groups.

In the 2nd week of the study, the application of Activitiy-2 was performed in the experiment group. Activity-1 was carried out as a big class discussion in a U-shaped seating arrangement. Since the discussion was at the beginning of the subject and the fact that students had never experienced such an application, a short briefing was given about the culture of discussion. Students were asked to freely express their responses to the questions and their corresponding justifications without any restrictions. Based on these responses, students were allowed to speak their minds until 5 different arguments were discovered. An effort was made so that students' responses included more than one justification. In order to achieve this, various conversational strategies such as listening, asking for opinion, asking for explanation, requiring example/evidence, reformulation and diversification of ideas, and different conversation tools ("tell me more", "why?", "who wants to add something?", etc.) were employed.

At the beginning of Activity-2, the 5 different arguments and their justifications, which were discovered in the big in-class discussion during Activity-1, were summarized on the blackboard by the researcher. During this summary, the researcher asked students to verify, and if necessary correct, their arguments. Then, students were tasked to write their arguments down in the activity sheet. After completing this task, students were informed that they would be attending another big in-class discussion and they were required to compare arguments during this discussion. While comparing the arguments, students were asked to think about each argument whether it sounded logical, and if the justifications for argument were strong or weak. In order to compare arguments and create their own responses, students were given enough time and made share their comparisons. The researcher utilized numerous conversational strategies such as fusing ideas, asking for ideas, asking for explanations and diversifying the ideas, and different conversation tools ("do you agree/disagree?", "why?", "who wants to add something?", "who can repeat?", etc.). When it was observed that the desired rebuttal of ideas started to emerge and the students arrived at the required decision, the comparison discussion was concluded. Writing down the arguments with their corresponding justifications during the summary, made determination and elimination of students' misconceptions easier. Since the aim was to enable students to enable students to compare each other's arguments and take advantage of numerous rebuttals, binary conclusions such as wrong/correct were absolutely avoided. Even the most problematic comparisons were kept for decision-making stage.

Upon completion of these two activities, a big group discussion was held to facilitate decision-making. The correct arguments that were discovered by the students were repeated back by the researcher and written down to the blackboard. The students were also asked to note these arguments. Feedback about the misconceptions, which were expressed by the students during the comparison stage, and roots of these misconceptions was provided. The mistakes made by the students were clearly communicated and discussions were arranged in order to enable students to make comparisons between the correct expressions and their incorrect expressions. During the comparison of student responses, a positive atmosphere was tried to be created as much as possible and it was emphasized that every single response is extremely valuable.

In the 4th week of the study, concept cartoons were applied to both experiment and control groups. Student responses were then graded and feedback was provided in order to eliminate observed misconceptions. Students' questions about the grades were discussed in the class and they were made feel that they are part of a mutual learning process. In the context of the study, the same exercise pages and practice materials were used in both experiment and control groups during 4th and 5th week.

DATA ANALYSIS

Prior to the analysis of data, which was gathered during the research, it was examined for lost data and outliers. There found to be no missing data within the data set. Participant No 48 was discovered to be an outlier and thus removed from the data set. Consequently, the normality of the test score distributions of both experiment and control group were investigated. Corresponding results are represented in Table 3.

Table 3. Tests of Normality

Score	Experiment Group			Control Group		
	Shapiro-Wilk			Shapiro-Wilk		
	Statistic	df	Sig.	Statistic	df	Sig.
Anxiety Pre-test	,951	27	,226	,973	28	,667
Anxiety Post-test	,927	27	,057	,965	28	,460
Success Pre-test	,932	27	,076	,955	28	,259
Success Post-test	,891	27	*,009*	,968	28	,517
Anxiety Pre-test_F1	,872	27	*,003*	,982	28	,900
Anxiety Post-test_F1	,786	27	*,000*	,988	28	,984
Anxiety Pre-test_F2	,941	27	,127	,894	28	*,008*
Anxiety Post-test_F2	,890	27	*,008*	,893	28	*,008*
Concept cartoon	,925	27	,052	,902	28	*,013*
Difference F1	,901	27	*,014*	,906	28	*,015*
Difference F2	,954	27	,262	,640	28	*,000*

When Table 3. is examined, it can be observed that anxiety pre-test and post-test grades and success pre-test grades were normally distributed for both groups. Therefore, parametric techniques were employed when these results were being analyzed. For other grades that are present in the table, non-parametric techniques were used.

For each grade, descriptive statistics were calculated. In order to determine the impact of dialogic teaching application on students' mathematics anxiety, two-factor variance analysis was applied to single-factor repetitive measurements, and to examine its effects on academic success (concept cartoon graded according to rubric with the success test), Wilcoxon Signed Ranks Tests and Mann Whitney U Tests was utilized.

Content analysis was used for qualitative data, which was obtained via the video recordings of the lectures, Recordings were watched twice by the researchers and time ranges, in which dialogs that are related to the 5 stages of dialogic teaching occurred most frequently, were determined. While representing the qualitative data, code "T" for teacher and codes "S1, S2, S3,…" for student were assigned. Moreover, students' concept cartoon grades were analyzed and presented in a supplementary manner to other findings.

The validity of the study was ensured with expert opinion, participant confirmation and detailed descriptive methods. The reliability was secured by confirmation and consistency investigations. In this study, in order to increase internal validity, diversification was chosen while collecting data. Additionally, description was utilized to further contribute to validity. In detailed description, as much detail as possible was tried to be retained while remaining true to the nature of raw data (Yıldırım and Şimşek, 2013). In order to increase the internal reliability of the study, findings from the video recordings were presented with direct citations.

FINDINGS

First the significance of the difference between pre-test grades of experiment and control group participants were examined. The results of the independent samples t-test is presented in Table 4.

Table 4. Difference between Pre-test Grades of Experiment and Control Group participants-1

	Group	N	Mean	Std. Deviation	t	df	Sig.
Anxiety Pre-test	Experiment	27	51,19	15,711	,894	53	,375
	Control	28	54,89	15,054			
Success Pre-test	Experiment	27	3,63	2,078	,377	53	,708
	Control	28	3,43	1,874			

When Table 4 is investigated, it can be seen that the difference between pre-test grades of experiment and control group participants was not statistically significant ($p>0.05$). This result indicates that the mathematical anxiety levels and existing knowledge about the subject for students in both groups were similar. Under these circumstances, these grades were used while determining the efficacy of the experimental process.

Table 5. Difference between Pre-test Grades of Experiment and Control Group participants-2

	Group	N	Mean Rank	Sum of Ranks	U	Sig.
Mathematics Learning Anxiety Pre-Test	Experiment	27	22,39	604,50	226,500	,011
	Control	28	33,41	935,50		
	Total	55				
Mathematics Evaluation Anxiety Pre-Test	Experiment	27	21,59	583,00	205,000	,004
	Control	28	34,18	957,00		
	Total	55				

After examining Table 5, the difference between pre-test grades for the anxiety sub-dimension of experiment and control group participants were found to be statistically significant ($p < 0.05$). This finding points to the fact that the grades of experiment and control group students about mathematical anxiety sub-dimension was not similar. As a result, while testing the efficacy of the experimental process, the difference between pre-test and post-test grades were used.

The descriptive statistics about before and after measurements of mathematical anxiety for the students in the study group were depicted in Table 6.

Table 6. Descriptive Statistics

	Group	Mean	Std. Deviation	N
Anxiety Pre-test	Experiment	51,19	15,711	27
	Control	54,89	15,054	28
	Total	53,07	15,351	55
Anxiety Post-test	Experiment	47,96	14,973	27
	Control	59,61	15,140	28
	Total	53,89	16,033	55

When Table 6 was examined, a decrease in the anxiety grades of experiment group students from pre-test to post-test was observed, whereas, in the control group, an increase in the anxiety grades of the students from pre-test to post-test was spotted. In order to determine if this differentiation of grades obtained from the anxiety scale of two groups of students, one of whom was subjected to the experimental process and the other one was not, is statistically significant, in other words, gauge the efficacy the experimental process on the total grades obtained from the anxiety scale, the results of a two-factor variance analyses (Two-Way ANOVA for Mixed Measures) was used for single-factor repetitive measurements was presented in Table 7.

Table 7. The efficacy the experimental process on the total grades obtained from the anxiety scale

Source	Sum of Squares	df	Mean Square	F	Sig.	Partial Eta Squared
Between-subjects						
Group	1619,760	1	1619,760	3,872	,054	,068
Error	22172,204	53	418,343			
Within-Subjects						
factor1	15,300	1	15,300	,340	,562	,006
factor1 * Group	432,900	1	432,900	9,631	,003	,154
Error (factor1)	2382,190	53	44,947			

When Table 7 was studied, it was seen that for both experiment and control group students, the difference in anxiety levels before and after the experiment is statistically significant. In other words, being in different process groups and taking repetitive measurements in different times exhibited a statistically significant mutual impact students' mathematical anxiety levels ($F_{(1, 53)}$= 9.631, $p<0.05$, η^2=0.154). This finding indicates that the change in mathematical anxiety from pre-test to post-test of the students, who were subjected to dialogic teaching (experiment group), was different from the students, who were in the control group. In short, mathematical anxiety of experiment and control groups differed according to the applied experimental process. Mathematical anxiety changes as a result of this application. This change in the mathematical anxiety of the students arises from the fact that dialogic teaching was used during education. As a result, using dialogic teaching instead of conventional methods during education is an important factor for decreasing mathematical anxiety of students.

The results of Mann Whitney U test, which was conducted to determine the significance of the difference between pre-test and post-test grades of mathematical learning anxiety sub-dimension and mathematical evaluation anxiety sub-dimension, were presented in Table 8.

Table 8. The significance of the difference grades

	GROUP	N	Mean Rank	Sum of Ranks	U	p
Mathematics Learning Anxiety Pre-Test	Experiment	27	16,28	439,50	61,500	,000
	Control	28	39,30	1100,50		
	Total	55				
Mathematics Evaluation Anxiety Pre-Test	Experiment	27	20,69	558,50	180,500	,000
	Control	28	35,05	981,50		
	Total	55				

When Table 8 was examined, for the students, who were subjected to dialogic teaching application (experiment group), the difference between pre-test and post-test grades of mathematical learning anxiety ($\bar{X}$=16.28) and mathematical evaluation anxiety ($\bar{X}$=20.69) was lower in a statistically significant manner than the difference between pre-test and post-test grades of mathematical learning anxiety ($\bar{X}$=39.30) and mathematical evaluation anxiety ($\bar{X}$=35.05) of the students, who were educated about the same material by using conventional techniques ($p<0.05$). These results point to the fact that dialogic teaching in mathematics courses did decrease mathematical learning anxiety and mathematical evaluation anxiety for the sub-learning field of continuity within the subject of limit and continuity.

The average values and their corresponding standard deviations of both experiment and control groups students' RMARS sub-dimension grades can be found in Table 9.

Table 9. Descriptive Statistics

Group	Factor	Pre-Test			Post-Test		
		N	$\bar{X}$	S	N	$\bar{X}$	S
Experiment	F1. Mathematics learning anxiety	27	29.74	9.13	27	25.93	7.57
	F2. Mathematics evaluation anxiety	27	21.44	8.65	27	17.44	6.33
Control	F1. Mathematics learning anxiety	28	37.32	12.41	28	41.14	12.10
	F2. Mathematics evaluation anxiety	28	28.61	8.64	28	29.32	7.96

When Table 9 was investigated, RMARS average F1 and F2 values of experiment group students decreased from beginning to the end of dialogic teaching application. Whereas, average RMARS F1 and F2 values for the control group had increased between pre-test and post-test.

It was established in Table 4 that students' existing knowledge about continuity sub-learning domain of limit and continuity subject was not

statistically significant. The descriptive statistics about students' pre and post experiment mathematical successes can be observed in Table 10.

Table 10. Descriptive Statistics

	Group	Mean	Std. Deviation	N
Success Pre-test	Experiment	3,63	2,078	27
	Control	3,43	1,874	28
Success Post-test	Experiment	14,74	3,526	27
	Control	7,57	1,874	28

After studying Table 10, it can be seen that for both experiment and control group students' success score averages, there is an increase from pre-test to post-test. This increase is more pronounced in the experiment group. In order to determine whether this difference in improvement is statistically significant, which also means determining the effectiveness of the experiment on mathematical success, Wilcoxon Signed Ranks and Mann Whitney U tests were conducted and their results are presented in Table 11 and Table 12 respectively.

Table 11. Difference between success Pre-test and success post-test of the participants

		N	Mean Rank	Sum of Ranks	Z	Sig.
Experiment Group Post-Pre	Negative Ranks	0[b]	,00	,00	4,554	,000
	Positive Ranks	27[c]	14,00	378,00		
	Ties	0[d]				
	Total	27				
Control Group Post-Pre	Negative Ranks	0[b]	,00	,00	4,647	,000
	Positive Ranks	28[c]	14,50	406,00		
	Ties	0[d]				
	Total	28				

b. post < pre, c. post > pre, d. post = pre

When Table 11 was examined, it was observed that the difference in mathematical success between pre-test and post-test for both experiment ($Z=4.554$, $p<0.05$) and control group students ($Z=4.647$, $p<0.05$) regarding the continuity sub-domain of limit and continuity subject was indeed statistically significant. This difference is in favor of positive ranks, which means on both groups, students' post-test scores were higher than their pre-test scores.

Table 12. Difference between Post-test Grades of Experiment and Control Group participants

	GROUP	N	Mean Rank	Sum of Ranks	U	p
Mathematics success	Experiment	27	40,04	1081,00	53,000	,000
	Control	28	16,39	459,00		
	Total	55				

After investigating Table 12, experiment group students' post-test mathematical success rank average ($\bar{X}=40.04$) is higher than control group students' mathematical success rank average ($\bar{X}=16.39$) in a statistically significant manner ($U=53.000$, $p<0.05$). These results point to the fact that dialogic teaching in mathematics courses increased mathematical success of students for the sub-learning field of continuity within the subject of limit and continuity.

Within the context of the study, the arguments that student created, which were conformant to the nature of dialogic teaching, were tried to be revealed through a concept cartoon. Difference between concept cartoon grades of experiment and control group participants shown in Table 13.

Table 13. Difference between concept cartoon grades of experiment and control group participants

	Group	N	Mean Rank	Sum of Ranks	U	p
Concept cartoon	Experiment	27	38,59	1042,00	92,000	,000

	Control	28	17,79	498,00		
	Total	55				

When Table 13 was examined, experiment group students' concept cartoon grade rank average ($\bar{X}$=38.59) is higher than control group students' concept cartoon grade rank average ($\bar{X}$=17.79) in a statistically significant manner (U=92.000, $p<0.05$). This result is indicative of the fact that experiment group students offered more arguments and justifications than control group students and it shows the root cause for these arguments and justifications were dialogic teaching, which used as the interference.

FINDINGS RELATED TO THE STAGES OF DIALOGIC TEACHING PROCESS

In this section, all results that were obtained via video recordings of each stage of dialogic teaching were presented in an ordered structure.

RESULTS RELATED TO PROBLEM INTRODUCTION AND ARGUMENT DISCOVERY

The results related to the problem introduction and argument discovery, which are the first 2 stages of dialog teaching, are presented in Table 14.

Table 14. Summary of Problem Introduction and Argument Discovery Stages of Dialogic Teaching

In-Class Conversations	Nature of Mathematics and Theories	Conversational Stragegies / Moves
...What can you say about the limit values of May and June graphs of bacteria population with respect to lake population? (T)	Developing alternate solution to the problem	Asking for explanation
Right and left limit values are different. Not for May (S3)	Offering hypotheses	-
What is you friend trying to say? Can you explain? (T)	-	Asking for explanation
Yes for May, but No for June. Because, for limit to exist, both right and left limits should be equal. It converges to the same value in June, Yes, it exists,,, (S5)	Justification	-
What do you say? Do you agree with your friend? (T)	Revealing thoughts via dialogy	Asking for opinion /Elaborate
According to the graph, if there was no disinfection, the increase would have continued but since there is a discontinuity in the graph, there is no limit. The increase was not continuous, it was discrete… (S1)	First argument/hypothesis and Justification	
So you mean if there is no limit around a point, the function is continuous at that point. OK, do you say whether since there is no limit, it is not continuous, or since it is not continuous, there is no limit? (T)	Revealing thoughts via dialogy	Reformulation / Diversifying ideas
… we said the limit exists for the month of June. Since it converges to the same value both from right and left, I say there is continuity. Because the graph has separate parts in May, there is no continuity. (S8)	Second argument/hypothesis and Justification	
...so you say there is no continuity since it's interrupted… (T)	Revealing thoughts via dialogy	Reformulation
It is important whether it is interrupted or not. Here, some are empty and some are full. When it is empty, it is undefined… (S5)	Looking for another argument	
… I hear opinions saying that if there are more interruptions, we cannot talk about continuity at that point. If you compare the limits and values for three	Revealing thoughts via dialogy	Reformulation / Diversifying ideas

In-Class Conversations	Nature of Mathematics and Theories	Conversational Strageties / Moves
points considering these ideas, what kind of relationship do limit and continuity have? (T)		
… Right and left limits are different when it noncontinuous. But, we can say there is limit where it is continuous. For others, limit exists but, since it is undefined , it is not continuous… Here, right and left limits are different, it is defined but it is noncontinuous nonetheless… (S10)	Third argument/hypothesis and Justification	
What did your friend try to say? (T)		Asking for explanation
For it to be continuous at a point, it must have a limit and it must be defined at that point. (S14)	Third argument/hypothesis	
Do you agree with this idea? (T)	Looking for another argument	
But right and left limits are different. Hence, even though it is defined at that point, it is not continuous. (S8)		
What do you think? (T)	Looking for another argument	Diversifying ideas
It makes sense to me, too. Although right and left limits are equal, when its value is different, it is noncontinuous. Therefore, right limit, left limit and the function value should be equal… (S4)	Fourth argument/hypothesis and Justification	
Any other ideas? (T)	Looking for another argument	Asking for explanation

Video: Between 13th and 34th minutes

When Table 14 was examined, it is observed that the thoughts of the students were encouraged to form justification and as a result their thoughts were drawn out in a dialogical manner by utilizing conversation moves such as asking explanation for hypotheses, diversification of ideas. This stage was finalized after students discovered 4 different arguments about the concept, which fullfilled the requirements for advancing to the next stage.

RESULTS RELATED TO ARGUMENT SUMMARY AND COMPARİSON STAGES

After identifying the problem and presenting the arguments, the experiment advances to argument summary and comparison stage. At this stage, with the help of reinforcing discussions, students were asked questions, which directed them to argument comparison. The results of Dialogic Teaching's argument summary and comparison stages were presented in Table 15.

Table 15. Summary of Dialogic Teaching's argument summary and comparison stages

In-Class Conversations	Nature of Mathematics and Theories	Conversational Strategies / Moves
… I guess there no more ideas other than these 4 so, let's write them down (T)	Fusing of ideas with teacher's scaffolding	-

In-Class Conversations	Nature of Mathematics and Theories	Conversational Strategies / Moves
1. If a function's graph is discontinued at a point, it cannot be continuous at that point 2. If the limit does not exist at a point, the function cannot be continuous at that point 3. In order for a function to be continuous at a point, its limit must exist and the function should be defined at that point 4. For continuity, right and left limits should exist, and the function should be defined at a point. And all three should be equal.		
Let's continue the discussion based on these ideas and try to reach some conditions for continuity by mathematically evaluating these situations in the light of our second activity	Evidence-based scientific reasoning	Asking for evidence-based scientific reasoning
[Students are thinking and talking among themselves]		
Now let's discuss about these ideas. Everyone can compare their ideas with others and express their opinions… (T)		Asking for opinion
My friend said, a graph is either continuous or not, however, we consider a certain point for continuity. If there is an interruption, it's not continuous. Therefore, the first argument is true. It can be discrete even the limit exists. Let's recognize that. So, if there is limit but also there is interruption, there is no continuity. I agree with the first two arguments but there are missing points. (S2)	Mutual knowledge generation of peers through comparisons	Argument comparison
Why do you think there are missing points? (T)		Asking for explanation
… we saw that if the value and the limit is not equal, there is no continuity. So, being defined is not enough. Hence, we can talk about three conditions for continuity. I say 4. That is most comprehensive one. (S9)	Argument, justification and rebuttal	Argument comparison
… we saw that limit should exists and only if it is equal to the value of the function at that point, the function is continuous. Therefore, for continuity, all three of them should be equal and in that case the most correct argument is the fourth one, right? (S5)	Argument, justification and rebuttal	Argument comparison
From a scientific point of view, the fourth argument can be accepted for continuity. We saw noncontinuous functions even they had a limit at a point… (T)	Argument, justification and rebuttal	Argument comparison

Video: Between 5th and 27th minutes

After investigation Table 15, it was seen that students exercised evidence-based reasoning and feedback about their misconceptions and root cause of these misconceptions were given to them during argument comparisons. This stage was ended when the justified argument has been discovered among compared ideas, which fullfilled the requirements of advancing to the next stage.

RESULTS RELATED TO DECISION-MAKING STAGE

The findings obtained in the last stage of Dialogic Teaching, namely decision-making, are presented in Table 16.

Table 16. Summary of Dialogic Teaching Decision-Making Stage

In-Class Conversations	Nature of Mathematics and Theories	Conversational Strategies / Moves
Then, who would want to explain the result we reached? Let's write it on the blackboard. (T)	Dialogically agreed upon results are obtained	Asking for explanation
So, in order to be continuous at a point, limit to exist is a precondition. Moreover, the function should be defined at that point. And it needs to have the same value as its limit. This is the result we all agree after discussion, correct? (S7)	Scientific decision-making	Decision
Then, let's write the conclusion we reached on the blackboard. A function, which is continuous at a point, has also a limit at that point. However, not every function having a limit, needs to be continuous. We can say that every argument helped us reaching this conclusion. Thank you. You can write down the reached conclusion. (T)	Scientific decision-making	Decision

Video: Between 3rd and 16th minutes

When Table 16. was examined, it is concluded that, conformant to the last stage of dialogic teaching application, students had reached a result, which they agreed upon.

DISCUSSION AND INTERPRETATION

In the light of findings of the study, it is concluded that dialogic teaching application has increased the academic successes of 12th grade students in the sub-learning domain of continuity. Furthermore, it was observed that the student in the experiment group, where dialogic teaching application was performed, have formed more arguments and justifications than the student in the control group. There are many studies conducted on the factors affecting the academic success of student in the field of mathematics and the efficacy of methods that were geared towards increasing academic success (Garfield and Ahlgren, 1988; Stylianides and Stylianides 2007; Özturan-Sağırlı, Kırmacı and Bulut, 2010; Cansız, 2015; Şahin, 2016). While this study shows some similarity with respective research pattern, it distinguishes itself with the application of dialogic teaching. Moreover, the results of the study is in alignment with other studies, whose subject were the effect of dialogic teaching conformant curricula on students and their academic success (Applebee et al., 2003; Juzwik et al., 2008; Güneş, S. 2013; Şahin, 2016). In this context, by showing the improving effect of the education process, which was prepared according to the five stages of dialogic teaching, on the mathematical success within the sub-learning domain of continuity, the study contributes to the literature.

From the quantitative results of the study, it can be concluded that by using the skills driven by dialogic teaching such as development alternative solutions, creating arguments, justification of those arguments, evidence-based scientific reasoning, investigation of validity and reliability of the evidence and reaching scientific decisions, students learned concepts more effectively. Furthermore, it was observed that students participated discussions more actively, created arguments that are in line with the nature of mathematics and developed several justifications for those arguments. This result is in line with the conclusion that scientific justification (argumentation) has a reinforcing effect on students' newly learned concepts (Şahin, 2016), creating a positive class atmosphere (Applebee et al., 2003) and it enables effective learning by making understanding the true nature of mathematics easier (Garfield and Ahlgren, 1988; Güneş, 2013).

It is evident that dialogic teaching application has decreased the mathematical learning anxiety and mathematical evaluation anxiety of 12th grade students in the sub-learning domain of continuity. This finding is in alignment with the results of studies, where students had high levels of mathematical anxiety (Richardson and Suinn, 1972; Tobias and Weissbrod, 1980) and these high levels of anxiety affected mathematical success and, via various teaching designs, these anxiety levels could be reduced (Klausmeier and Goodwin 1971; Richardson and Suinn 1972; Betz 1978). Within this context, by demonstrating the fact that an education process, which was designed in accordance with the five stages of dialogic teaching, could reduce mathematical anxiety, this study makes a contribution to the field.

When the qualitative findings were examined, it can be observed that with dialogic teaching, it was possible for students to developed alternative solutions to a problem, propose arguments, justify those arguments, perform evidence-based scientific reasoning and draw scientific conclusions. This helped them to understand concepts deeper, improve mathematical success and decrease their mathematical anxiety. These results are similar to the results of studies that analyzed in-class conversations regarding the fact that students deriving new arguments from conflicting ideas within a dialogic teaching based education (Reznitskaya, Anderson and Kuo, 2007; Juzwik et al., 2008; Güneş, 2013; Şahin, 2016). Moreover, these findings are in alignment with other research results, which conclude that dialogic teaching can reduce anxiety by creating a more positive and collaborative class atmosphere (Applebee et al., 2003) and by utilizing certain strategies that are geared towards problem-solving with evidence-based scientific reasoning, which makes mathematical learning an easier process (Garfield and Ahlgren, 1988; Stylianides and Stylianides, 2007).

Since this research is limited to a certain level of education and a certain subject, in order to contribute to the literature and the development of curricula, it will be beneficial to conduct similar studies on the effects of dialogic teaching if it was applied in other courses and its relationship with other factors. If teachers use this teaching application, they can overcome the pedegogical problems they face in the classroom. It is suggested that teachers should be taken into consideration in order to be more effective in the teaching process, since the creation of these concepts also provides a basis for subsequent concepts.

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